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NOTES

ON

THE THEORY OF STRUCTURES

by

ilton
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REACTIONS——MOMENTS——SHEARS——MOVING LOADS——BEAMS——GIRDERS——
SIMPLE TRUSSES.

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CHAPTER I.

OUTER AND INNER FORCES.

1. DEFINITIONS. A structure as defined in the "Century Dictionary" is a production or piece of work artificially built up, or composed of parts joined together in some definite manner. As used in this book, however, the word will be restricted in its meaning to a part or combination of parts constructed to hold IN EQUILIBRIUM definite forces, with special reference to bridges and buildings.

Structures may be either STATICALLY DETERMINED or STATICALLY UNDETERMINED. Statically determined structures are those in which the reactions and primary stresses can be computed by statics. Structures for which these functions cannot be obtained by statics belong to the second class.

A BRIDGE is a structure built to provide transportation across some natural or artificial obstacle such as a river, ravine, street or railroad. The term includes not only the superstructure of wood, metal, or masonry but also the substructure which may consist of masonry piers and abutements, or of steel towers. The superstructure may consist of simple beams supported at the ends directly on the masonry, or in case of long spans supported on cross beams which are themselves supported at the ends by girders, trusses or arches. In the latter case the longitudinal beams are known as STRINGERS and the cross beams as FLOOR BEAMS. The combination of stringers and floor beams is usually referred to as the bridge floor. As a clear conception of the function of the stringers and floor beams is essential to the understanding of the matter which follows, the student is advised to study carefully Figs. 1a and 1b, 1c and 1d and to examine some of the bridges in his vicinity.

19 March 1927

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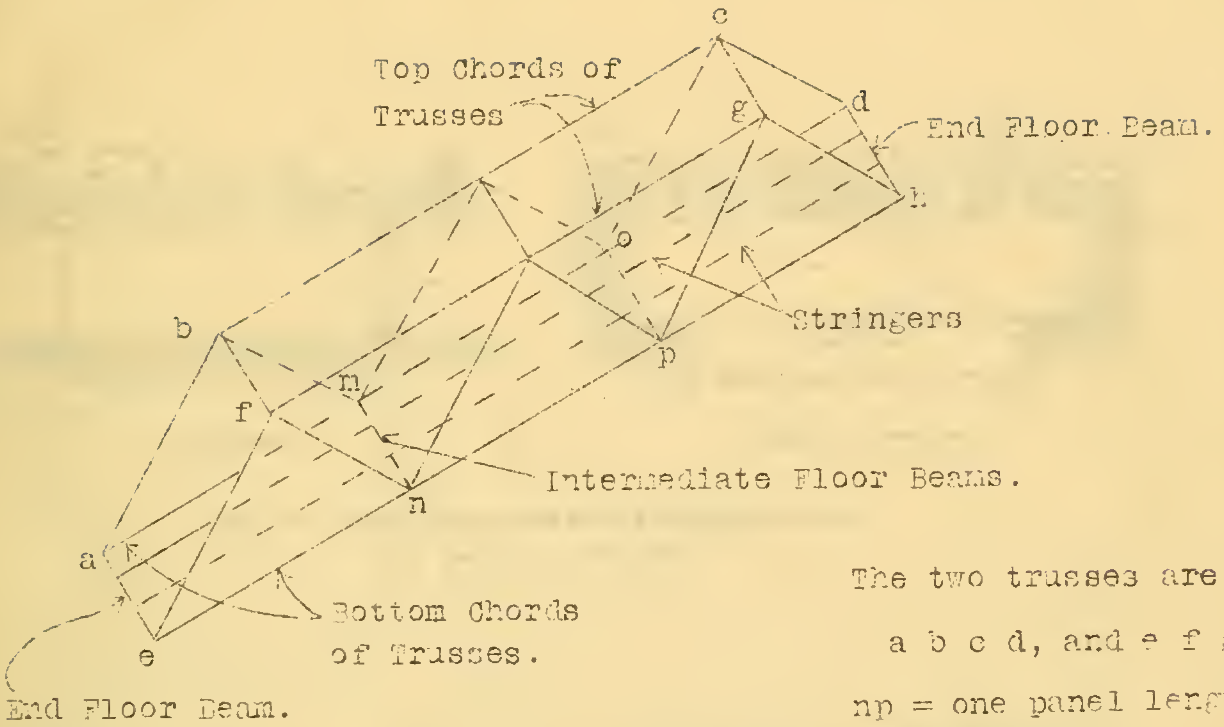


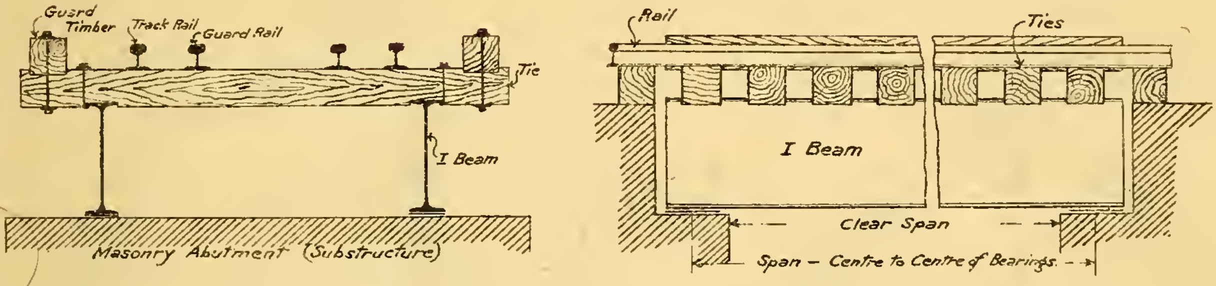
The two trusses are
a b c d, and e f g h.
np = one panel length.

Fig. 1a.

Sketch Showing Steel Work of a Single Track R. R. Bridge.

All bracing is omitted from sketch for clearness. Stringers are separate beams supported at each end on the floor beams which are in turn supported on the trusses which carry the loads to the abutments. The trusses are supported on the abutments at points a, e, h, and d. The abutments are not shown in the sketch.

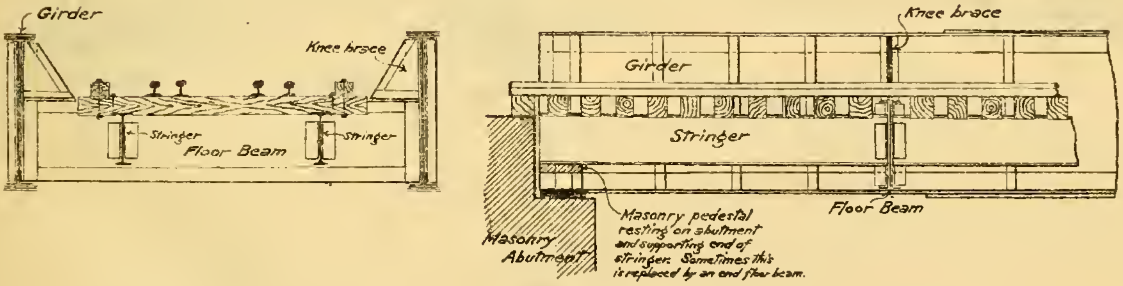
I BEAM AND PLATE GIRDER BRIDGES.


Guard Timber
Track Rail
Guard Rail
Tie
I Beam
Masonry Abutment (Substructure)
Rail
Ties
I Beam
Clear Span
Span – Centre to Centre of Bearings.

END. VIEW.

SIDE ELEVATION.

FIG. 1b I BEAM BRIDGE FOR SINGLE TRACK RAILROAD.
(A DECK STRUCTURE.)


Girder
Knee brace
Stringer
Floor Beam
Stringer
Knee brace
Girder
Stringer
Masonry Abutment
Masonry pedestal resting on abutment and supporting end of stringer. Sometimes this is replaced by an end floor beam.
Floor Beam

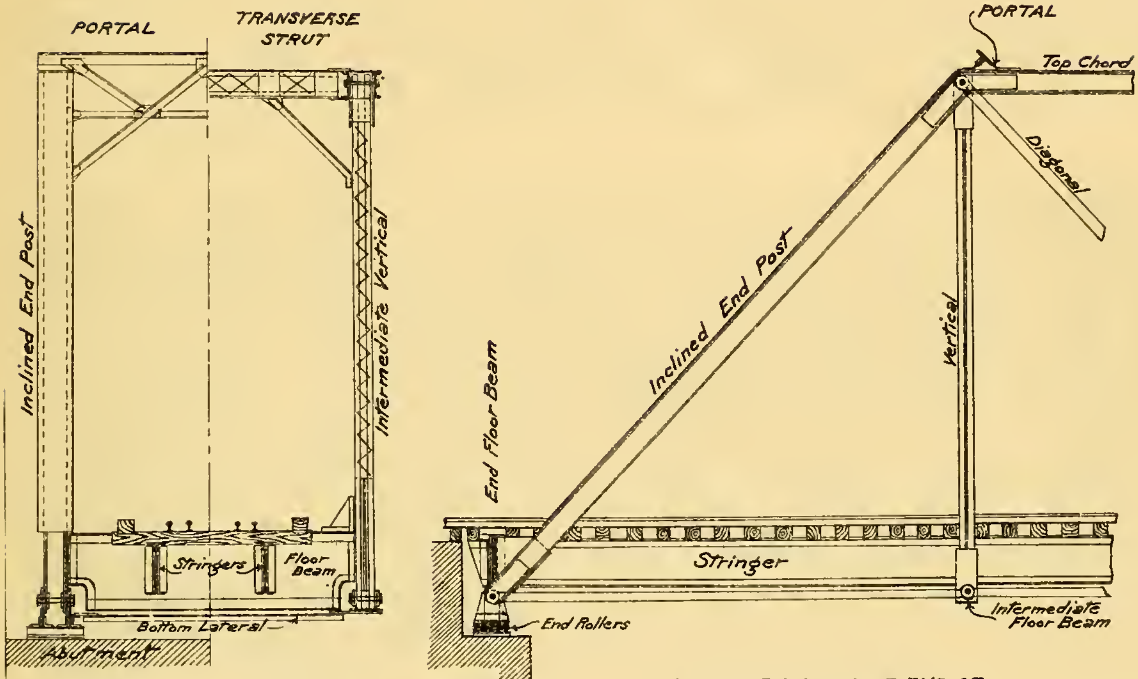
CROSS SECTION.

LONGITUDINAL SECTION.

FIG. 1c HALF-THROUGH SINGLE TRACK
PLATE GIRDER RAILROAD BRIDGE.

NOTE:-
Portion of bridge between floor beams (measured along bridge) is called a panel.

PORTION OF A SINGLE TRACK THROUGH RAILROAD BRIDGE.



HALF END VIEW.

HALF CROSS SECTION.

SIDE ELEVATION OF END OF A PIN CONNECTED TRUSS.

FIG. 1^d

Bridges may be either through, deck, or half-through. Through bridges are those the depth of which is sufficient to permit the use of overhead cross bracing. In deck bridges the superstructure is below the roadway and in short railroad spans the floor systems is sometimes omitted, the ties being supported directly on the top of the girders or trusses. Half-through bridges are those in which the floor is supported at an elevation below the top of the supporting girders or trusses, but which have insufficient depth to permit the use of overhead bracing. In such bridges the top chords of the girders or trusses are braced laterally by brackets or knee braces instead of transverse bracing.

Whether a deck or through bridge should be used for a given location depends upon the external conditions. In general, bridges of considerable span are built as through structures unless the approaches on either side are at a considerable elevation above the obstacle to be crossed. The solution of this question for a given case is usually obvious and will not be considered here.

2. FORCES. The forces acting upon a structure may be divided into two classes; outer and inner. The outer forces consist of the applied loads and the resultant reactions, and may be divided into two distinct types; live or moving loads, and dead or quiescent loads. The inner forces are the molecular forces which are brought into action by the outer forces and hold them in equilibrium. The dead load includes the weight of the structure itself, and all of its permanent quiescent load such as the pavement on highway bridges; the rails, and other track appurtenances on railroad bridges; the floors, walls, roofs, and partitions in buildings. The live load consists of all forces which are applied intermittently. For bridges these may be locomotives and cars, vehicles, pedestrians, snow and wind; for buildings they consist of people, snow, wind, office furnishings and partitions.

3. OUTER FORCES. The determination of the intensity, distribution, and point of application of the outer forces is often difficult and requires mature judgment based upon extensive experience. For structures of great magnitude the question is particularly complicated and of vital importance; the design of such structures should never be attempted without a thorough study of this problem in its relation to the structure in question. In the following articles some of the difficulties in the way of an exact solution of the question will be presented and data given for use in the solution of the more common cases.

4. WEIGHT OF STRUCTURE. It is impossible to determine accurately the weight of a given structure before the completion of the design. It is equally impossible to design the structure with precision until its weight is known. It is therefore necessary in all cases to make use of approximate methods of solution, first assuming the weight, then designing with the assumed data, then computing the weight and revising the design in the light of the new information thus obtained. For the more common types of structure data accumulated by experience may be used by the

Bridges may be either through, deck, or half-through. Through bridges are those in which the roadway is carried at or near the level of the bottom chords. In deck bridges the roadway is placed on [illegible] the top chords [illegible] [illegible] bridges [illegible] the roadway [illegible] the [illegible] of the trusses [illegible] [illegible] include those in which the [illegible] below the top of the supporting girders or trusses, and which have [illegible] lateral [illegible] of overhead bracing. The [illegible] top chords of the girders or trusses are braced laterally by [illegible] or knee braces [illegible] of [illegible] bracing.

The proper type of [illegible] bridge should be [illegible] a given [illegible] upon the [illegible] conditions. In general, [illegible] as through structures [illegible] of [illegible] the [illegible] to be [illegible] The [illegible] will be [illegible] here.

1. **FORCES.** The forces acting upon a structure may be divided into two classes; dead and live. The dead forces consist of the weight of the structure and the resulting reactions, and may be divided into the structure proper [illegible] lateral bracing, floor beams, and deck or [illegible] The live forces are the transient forces which are brought into action by the moving loads and [illegible] in [illegible] The dead load includes the weight of the structure itself and all of the permanent [illegible] load such as the pavement [illegible] railway bridges, the rails, and other track [illegible] railroad bridges, the [illegible] and [illegible] in buildings. The live load consists of all loads which are applied intermittently. For highway bridges it may be [illegible] wagons, pedestrians, [illegible] and snow; for buildings [illegible] of people, snow, wind, [illegible] and [illegible].

2. **DEAD LOADS.** The determination of the intensity, distribution, and point of application of the dead forces is often difficult and requires mature judgment based upon extensive experience. For structures of great magnitude the question is particularly complicated and of vital importance; the design of such structures should never be attempted without a thorough study of this problem by the [illegible] the structure is [illegible]. In the following articles some of the difficulties in the way of an exact solution of the questions will be presented and [illegible] given [illegible] in the solution of the more common cases.

WEIGHT OF STRUCTURE. It is impossible to determine accurately the weight of a great structure before the completion of the design. It is equally impossible to design the structure with precision until its weight is known. It is therefore necessary in all cases to make use of approximate methods of solution; first assuming the weight, then, designing with the assumed data, then computing the weight and revising the design in the light of the new information thus obtained. For the more common types of structures [illegible] may be [illegible] by [illegible] designer and the first assumption made with sufficient accuracy to make revision unnecessary. For structures out of the ordinary and particularly those in which the weight of the structure itself is a large percentage of the total load, several revisions are sometimes necessary, and a final computation of the weight after the completion of the detailed drawings and before the commencement of shop work, should never be omitted. The failure to do this in the huge Quebec bridge which failed during erection in 1907, resulted in serious errors in the stresses for which the structure was designed.

In all cases the designer should first design completely the minor portions of the structure and determine their weight carefully so as to eliminate as much uncertainty as possible. For example, in the design of a railroad bridge, the stringers should first be figured and their weight carefully determined, the floor beams may then be designed, and then the lateral bracing, thus giving considerable information as to the total weight of the bridge and throwing the uncertainty into the main girders or trusses.

5. FORMULAS FOR WEIGHT OR RAILROAD BRIDGES. It is possible to estimate the weight of railroad bridges more accurately than other types of important structures since there is less variation in loads and conditions. Current practice on first class American railways differs but little and the following formulas may be used to obtain a preliminary estimate of the weight of bridges for such railways. These formulas are derived to fit the curves given for the weight of carbon steel bridges by Dr. J. A. L. Waddell in his paper entitled "Nickel Steel for Bridges" published in the "Transactions of the American Society of Civil Engineers." Vol. LXIII, pages 165 to 172. These bridges were designed in accordance with the Waddell specifications as given in "De Pontibus," John Wiley and Sons, publishers, and for two locomotives each weighing with tender 161.5 tons followed by a uniform load of 4600 lbs. per foot of track.

For weights of cantilever spans and nickel steel bridges the reader is referred to the original paper. For bridges designed for either heavier or lighter loads these weights may be altered in a somewhat less proportion than the live loads in accordance with the designer's judgment.

w = weight in pounds of steel per linear foot of bridge.
L = Span in feet between centres of end bearings.

Single-track deck plate-girder spans (	L=20ft. to 50 ft. $w=278+5.1L+0.1L^2$	5a
" " " " " "	L=50ft. to 70ft. $w=30+15L$	5b
" " " " " "	L=70ft. to 100 ft. $w=170+13.8L$	5c
" " half-through plate-girder spans.	L=30ft. to 70ft. $w=760+13L$	5d
" " " " " "	L=70ft to 120 ft. $w=574+16.8L$	5e
" " thro riveted spans.	L=100ft. to 115 ft. $w=1740$	5f
" " " " "	L=115ft. to 200 ft. $w=724+8.83L$	5g
Double " " " "	L=100ft. to 150 ft. $w=2115+3.5L+0.05L^2$	5h
" " " " "	L=150ft. to 200ft. $w=980+18.6L$	5i
Single " " pin-connected spans (Pratt-trusses)	L=170ft. to 370ft. $w=606+8.57L$	5j
Double " " " " " " "	L=180ft. to 350 ft. $w=1074+15.7L$	5k
Single " thro pin-connected spans (Petit-trusses)	L=350ft. to 600ft. $w=-108+10.88L$	5l
Double " " " " " " "	L=350ft. to 600ft. $w=1548+14.72L$	5m

6. TABULATED RAILROAD BRIDGE WEIGHTS. In order that the student may compare the weights given by above formulas with an independent set of values the following table has been prepared comparing some of the weights thus obtained with the weights of short span bridges furnished by Mr. A. M. Knowles, Assistant Bridge Engineer of the Erie R. R.

Table for Railroad Bridges. Weights per linear foot. Steel only.

Span	Character.	Wt. per ft. from formulas	Wt. per ft. Erie R.R. Designs.
20 ft.	Single-track deck plate girder	420	580
40	" " " " " "	642	700
60	" " " " " "	930	1000
80	" " " " " "	1274	1100
100	" " " " " "	1555	1400
40	" " " half-through plate girder	1280	1200
60	" " " " " " "	1540	1520
80	" " " " " " "	1918	1800
100	" " " " " " "	2254	1980
120	" " " " " " "	2590	2340
100	" " " through-riveted	1740	----
120	" " " " "	1784	2050
140	" " " " "	1960	2330
160	" " " " "	2137	----
180	" " " " "	2313	----
200	" " " " "	2490	----
175	" " " " pin-connected	2106	2480
200	" " " " " "	2320	----
250	" " " " " "	2746	----
300	" " " " " "	3177	----
350	" " " " " "	3601	----
350	" " " " " "	3702	----
400	" " " " " "	4244	----
450	" " " " " "	4792	----
500	" " " " " "	5332	----
550	" " " " " "	5882	----
600	" " " " " "	6420	----

The above figures do not give the gross weight. To obtain this add from 400 to 450 lbs. per foot depending upon the construction to allow for weight of track.

7. APPROXIMATE TRUSS WEIGHTS. Bridges differing materially from those previously considered and for which other data is not available may be estimated by the following rule devised by Clarence W. Hudson, Consulting Engineer, 45 Broadway, New York.

Let L maximum live stress in bottom chord.
" I impact in member in which L occurs.
" D_1 dead stress in same member due to known weight of floor.
" D_2 dead weight of truss and bracing (Guessed).
" f_t allowable unit stress in tension.
" A_1 area in square inches of member in which L occurs.
" A_2 " " " " per linear unit of one truss.
" W weight per linear foot of one truss and its bracing.

Then $A_1 \quad \frac{L + I + D_1 + D_2}{ft} \qquad A_2 = 5A_1$ and $W = \frac{50}{3} A_1$

The above is based upon an allowance of 1.25 A_1 for the upper chord, 1.25A_1 for the web members, A_1 for details, and .5A_1 for bracing. The weight of steel is used in round figures as 10 lbs. per sq. inch of cross section for a bar one yard in length.

This method is said to give a very close estimate of the actual weight.

8. WEIGHT OF HIGHWAY BRIDGES. The weight of highway bridges is less easily determined than that of railroad bridges since the width, span, floor covering and character of loading are subject to wide variations. Formulas have been deduced for special cases but these are of little value and will not be quoted. The designer should proceed step by step as previously stated and if experienced should obtain good results. The method given in Article 7 may be applied in order to obtain an approximate truss weight.

The following table is printed through the kindness of William Jackson, City Engineer of Boston, Mass., and may be used in determining weight of floor covering.

Weight of Paving Material.

NOTE;-
(B.M. = Board Measure)

Hard (yellow) pine ---4 lbs. per ft. B.M.-	48	lbs.	per	cub.	f
Creo-resinate yellow pine paving blocks.-	65	"	"	"	'
Spruce and white pine.-2 1/2 lbs.per ft.B.M.	30	"	"	"	'
Bricks - pressed and paving.-------------	150	"	"	"	'
Portland cement concrete.----------------	160	"	"	"	'
Tar concrete,(Base for Asphalt walks,etc.)	125	"	"	"	'
(Silician rock (Simpson Bros.)	140	"	"	"	'
Trinidad Asphalt (Barber Asphalt Co.)Refined	74				
as laid------------------------	140	"	"	"	
Granolithic or artificial stone,-------	150	"	"	"	
Pavements (exclusive of sand cushion)					
6" granite block-------------------	80	"	sq.	sq.	'
4" brick	50	per	sq.	foot.	
4" wood block (creo-resinate).-----	22	"	"	"	
Roadway waterproofing-------------------	12	"	"	"	

1 1/4 " thick (felt,roofing,pitch, sand and road pitch).

Buckle plates----------------10 lbs to 20 lbs per sq. ft.

9. WEIGHT OF ROOF TRUSSES. The weight of roof trusses depends upon the span, distance apart of trusses, roof covering, and roof pitch. The conditions are somewhat more uniform than for highway bridges and formulas , for approximate weights may be used with some degree of success.

The formula [*] which follows were deduced by N. Clifford Ricker and represents the weight of 50 roof trusses designed for spans from 20 ft. to 200 ft. in length. The distance apart of trusses varied from 10 ft. to 30 ft. and the rise from 1/10 to 1/4 the span. The roofs were assumed as covered with 7/8" wooden sheathing carrying painted tin. The snow load was taken as 20 lbs. per horizontal square foot and wind as 30 lbs. per square foot on a vertical plane, reduced to allow for pitch of roof. Trusses of yellow pine with steel verticals, of white pine with steel verticals, and entirely of steel were included.

Let W = weight of truss in lbs. per square foot of horizontal projection of roof.

Let S = span in feet.

$$W = \frac{S}{25} + \frac{S^2}{6000}$$

Steel trusses of shorter spans than 100 ft. probably weigh somewhat more than the value given by the formula, and white pine trusses are somewhat lighter than those of yellow pine.

For roof trusses under varying conditions, the value given by the formula may be modified or the designer may use his judgment.

For weights of various roof coverings see table which follow:-

The following table gives the approximate weights per square foot of roof surface for some of the roof coverings in common use. (See also latter portion of Article 15)

Pine shingles	2	lbs.
Corrugated iron, without sheathing	1 to 3.75	lbs
Felt and asphalt, without sheathing	2	"
Felt and gravel, without sheathing	5 to 10	"
Slate, 1/4" thick	9	"
Tin, without sheathing	.75	"
Skylight glass, 3-16" to 1/2", including frames,	4 to 10	"
White pine sheathing, 7/8" thick	2	"
Yellow pine sheathing, 7/8" thick	4	"
Tiles, flat	15 to 20	"
Tiles, corrugated	8 to 10	"
Tiles, on concrete slabs	30 to 35	"
Plastered ceiling	10 to 15	"

[*] See "Bulletin No. 16" of the University of Illinois, Engineering Experiment Station, entitled "A Study of Roof Trusses" by N. Clifford Ricker, D. Arch. Professor of Architecture.

10. WEIGHT OF STEEL OFFICE BUILDINGS. No attempt will be made to give data as to the weight of steel frame work of buildings of this character. The weights of walls, floors, partitions, and fire proofing form a considerable proportion of the weight of such structures, but these should be estimated in detail depending upon the character of construction as determined from the architects' plans.

The following values may be used to give idea of the weight of these portions of the building.

Maple floor 7/8" thick			Wt. per sq. ft.	4 lbs.
Brick wall per superficial sq.ft. Wt.per.in. of thickness of wall 9 1/2 to				10 "
Hollow tile floor arch.	Depth	8"	Wt.per.sq. ft.	27 "
" " " "	"	9"	" " " "	29 "
" " " "	"	10"	" " " "	33 "
" " " "	"	12"	" " " "	38 "
" brick for flat arches	"	6"	" " " "	27 "
" " " " "	"	7"	" " " "	29 "
" " " " "	"	8"	" " " "	32 "
" " " " "	"	9"	" " " "	36 "
" " " " "	"	10"	" " " "	39 "
" " " " "	"	12"	" " " "	44 "
" " clay,partition	Thickness	2"	" " " "	11 "
" " " "	"	3"	" " " "	14 "
" " " "	"	4"	" " " "	15 "
" " " "	"	5"	" " " "	19 "
" " " "	"	6"	" " " "	20 "
" " " "	"	7"	" " " "	
" " " "	"	8"	" " " "	27 "

For minimum weight of fireproof floors use 75 lbs. per sq. ft. and for office buildings add 10 lbs. per sq. ft. to this to allow for movable partition.

Porous terra-cotta partitions	Thickness	3"	Wt.per.sq. ft.	16 lbs.
" " " "	"	4"	" " " "	19 "
" " " "	"	5"	" " " "	22 "
" " " "	"	6"	" " " "	23 "
" " " "	"	8"	" " " "	33 "

Furring, Roofing and Ceiling.

Porous terra-cotta furring	Thickness	2"	Wt.per.sq. ft.	8 lbs.
" " " roofing	"	2"	" " " "	12 "
" " " "	"	3"	" " " "	15 "
" " " "	"	4"	" " " "	19 "
" " " ceiling	"	2"	" " " "	11 "
" " " "	"	3"	" " " "	15 "
" " " "	"	4"	" " " "	19 "

Concrete for building work may be made with cinders, broken stone, or gravel.

The weight of cinder concrete may be taken as 112 lbs. per cu. ft.

The weight of gravel may be taken as 150 lbs. per cu. ft.

" " " trap concrete or gravel concrete may be taken as 160 lbs. per cu. ft.

If concrete is reenforced with steel add 4 lbs. per cu. ft. to above weights.

Fireproofing for columns or beams may be either of terra cotta or concrete. The thickness should be not less than 2 inches. The weight per foot depends upon the size of the member to be protected.

11. LIVE LOADS FOR RAILROAD BRIDGES. It is possible to determine definitely the weights of the locomotives and cars used upon a given railroad. In consequence the actual live loads crossing a given bridge can be ascertained with considerable exactness though it is necessary to make due allowance for the effect of high speed, irregularities in track, and other dynamic effects which do not occur when the loads are at rest. These dynamic forces are considered in Articles 17 and 18 and will be neglected for the present.

In the design for a new bridge it is also desirable to make due allowance for future increase in weight of locomotives and cars, hence the loads for which bridges are designed may be somewhat heavier than those which are in actual use at the time of construction though the factor of safety (see Article 20) provides to some extent for such increase.

As to the type and number of locomotives and character of train loading, American practice is fairly uniform.

Two combinations are usually considered.

a. Two consolidation locomotives followed by a uniform load per foot.

b. A pair of axles with loads somewhat heavier than those of the consolidation engine and no uniform load.

The former loading gives the maximum for most cases but the latter is sometimes the controlling factor for stringers, short beam spans, and minor truss members.

In designing, the effect of rails and ties in distributing the locomotive load is usually neglected, the wheel loads being taken as applied at points.

As the actual variation in wheel spacing and loads for locomotives of different makes is often slight it has become in recent years the custom amongst many railroads to specify the typical locomotives, first proposed by Theodore Cooper, Consulting Engineer, of 45 Broadway, New York. In these locomotives the distance between axles are in even feet and the wheel loads in even thousands of pounds. While these loads and spaces may not represent actual cases they agree closely

Concrete for building [illegible] is made with 1:3:6 mixture, crushed stone, or gravel.

The weight of plain concrete may be taken at 145 lbs. per cu. ft.

The weight of gravel may be taken as 120 lbs. per cu. ft. [illegible] being composite in general character may be taken at [illegible] per cu. ft.

If concrete is reinforced with steel add 4 lbs. per cu. ft. to above weight.

Concerning the weight of Timber and the ability of being [illegible]. The [illegible] should be not less than [illegible] inches. The weight per lineal foot depends upon the size of the ties and [illegible] to be purchased.

10. LIVE LOADS FOR RAILROAD BRIDGES. It is possible to determine definitely the weights of the locomotives and cars used upon a given railroad. To construct the bridge for these loads assuming a given train [illegible] may be introduced with considerable variation though it is necessary to make some allowance for the effect of high speed, irregularities in track, and other dynamic effects which do not occur when the loads are at rest. These dynamic forces are considered in articles 11 and 12 and will be neglected for the present.

In the design of a new bridge it is also desirable to make due allowance for future increase in weight of locomotives and cars, [illegible] the [illegible] for such increase.

As to the type and number of locomotives and character of train loading, American practice is fairly uniform.

Two loadings are usually considered.

a). Two consolidation locomotives followed by a uniform load per foot.

b). A pair of excessively heavy axle loads somewhat heavier than those on the consolidation engine and at moderate spacing.

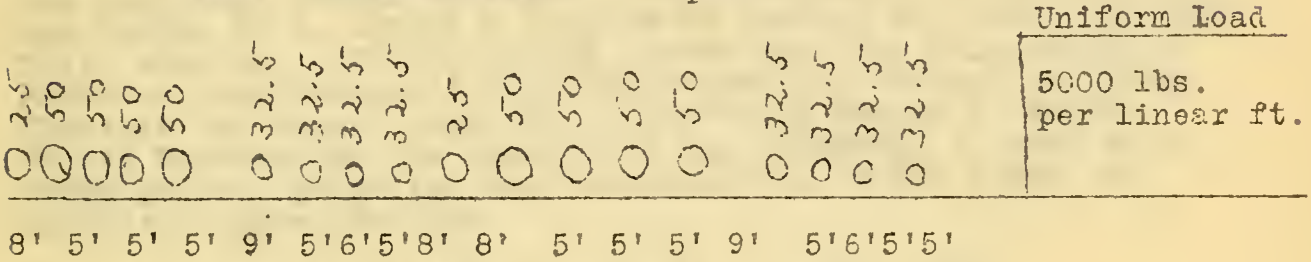
The former loading gives the maxima for most cases, but the latter is used where the controlling factor for stringers, short beam spans, and floor beam members.

In designing, the effect of cars and rail in distributing the [illegible] load is usually neglected, the wheel loads being taken as applied at points.

As the normal variation in engine loading [illegible] of different types is [illegible] Cooper's [illegible] loadings, first proposed by Theodore Cooper, consulting engineer, [illegible] the [illegible] between axle loads [illegible] and [illegible] with average locomotives and are much simpler to deal with than loadings in odd hundreds of pounds, and axle spacings in feet and inches. Moreover, the uncertain developments of the future and the unknown effect of impact make the use of such typical loads little less accurate than the use of actual engines.

Fig. 11 shows Cooper's E_{50} locomotive which is suitable for loads carrying heavy traffic.

Loads in thousands of lbs. per axle.



Cooper's Typical Locomotives, E_{50}

For other conditions types known as E_{60}, E_{40}, E_{30}, etc. are used, these differing from the E_{50} type in weight only; for example in the E_{40} type the driving wheel axle loads are 40000 lbs. instead of 50000 lbs., while the other loads are proportionate. This has the advantage of allowing tables of moments and shears made for one type to be readily used for another type by multiplying by a simpler ratio. Such tables are now incorporated in numerous hand books and specifications.

Before leaving the subject a few words as to other methods of loadings are desirable. Some years ago there was considerable agitation in favor of adopting a uniform load, in order to simplify computation. The advantage is obvious to those who are familiar with such work, but the disadvantage is that in order to obtain properly proportioned trusses this load must necessarily vary for different spans, and for different members in the same span. This offsets to a considerable extent the advantage gained. Moreover the adoption of the above standard loadings has further simplified the labor of computation for actual wheel loads so that at the present time it is believed that for ordinary structures the advantage of using a uniform load is too small to consider. For complicated trusses a combination of the two methods is probably best; viz: the use of wheel concentrations for web members and of a uniform load for the chords, since the approximation for the chords by using a uniform load is less than for the web members.

12. LIVE LOADS FOR HIGHWAY BRIDGES. The magnitude and character of such loads depends almost entirely upon the location of the bridge. If it be a large city or in a district where heavy manufactured articles, or quarried stones of great weight are to be transported it is quite probable that wagon loads from 20 to 30 tons may at times pass over the bridge. Electric carloads of from 40 to 50 tons should also be assumed as such cars are

with average locomotives and are more similar to deal with [illegible] in the matter of power, but with [illegible] in fuel and water. [illegible] [illegible] [illegible] [illegible] [illegible]

The following [illegible] the [illegible] locomotive which is suitable for [illegible] [illegible]:

[illegible]

[illegible]	[illegible]
[illegible]	[illegible]

[illegible]

[illegible]

The [illegible] [illegible] [illegible] [illegible]

14 [illegible]

already in use in some portions of the United States, while the construction of interurban electric lines in many sections of the country indicates a future wide spread extension of heavy street car traffic. The amount of foot travel and ordinary vehicular traffic on highway bridges is also something to be carefully studied. The weight per square foot from a crowd of people may reach the high figure of 150 lbs. which is probably heavier than the weight per foot from horses and wagons on the roadway. To assume, however, that such a load is likely to occur over the entire surface of an ordinary bridge is absurd, and the longer and wider the bridge the less the load that should be taken. In fact every considerable highway bridge is in itself a problem in loading and should be carefully studied. The following clause from The Massachusetts Railroad Commissioner's "Specifications for Bridges carrying Electric Railways," prepared by Professor George F. Swain, Bridge Engineer of the Commission may, however, be used as a broad general guide for the determination of the loads for ordinary highway bridges.

"(a) For city bridges, subject to heavy loads;-

"For the floor and its supports, a uniform load of 100 pounds per square foot of surface of the roadway and sidewalks, or a concentrated load of 20 tons on two axles 12 feet apart. In computing the floor beams and supports, the railway load shall be assumed, together with either (1) this uniform load extending up to within two feet of the rails, or (2) the above described concentrated load alone.

"For the trusses or girders, 100 pounds per square foot of floor surface for spans of 100 feet or less, 80 pounds for spans of 200 feet or over, and proportionally for intermediate spans. This uniform load is to be taken as covering the floor up to within two feet of the rails.

(b) For suburban or town bridges, or heavy country highway bridges;-

"For the floor and its supports, a uniform load of 100 pounds per square foot, or a concentrated load of 12 tons on two axles 8 feet apart; these loads to be used as described under (a).

"For the trusses or girders, 80 pounds per square foot of floor surface for spans of 100 feet or less, and 60 pounds for spans of 200 feet or more, and proportionally for intermediate spans; to be used as described under (a). (See d.)

"(c) For light country highway bridges;-

"For the floor and its supports, a uniform load of 80 pounds per square foot; this load to be used as described under (a). (See d.)

"For the trusses or girders, 80 pounds per square foot of floor surface for spans of 75 feet or less, and 50 pounds for spans of 200 feet or more, and proportionally for intermediate spans; to be used as described under (a)

"(d) All parts of the floor of a highway bridge should also be proportioned to carry a road roller weighing 15 tons, greater in case of some portions of the United States, while the concentration of population in certain cities of these sections of the country indicates a future when single vehicles of heavy weight are probable. The amount of such travel and heavy traffic on highway bridges is what cannot [illegible] to be accurately decided. The weight per square foot from a crowd of people may reach the high figure of 150 lbs., which is probably heavier than the weight per foot from heavy road wagons on the roadway. To assume, however, that such a load is likely to occur over the entire surface of an ordinary bridge is absurd, and the longer and wider the bridge the less the load that should be taken. In fact every municipality owning bridges is in itself a problem in loading and should be carefully studied. The following clauses from the Massachusetts Highway Commission's "Specifications for Highway Bridges, Standard Bridges," prepared by Professor George F. Swain, Bridge Engineer of the Commission may, however, be used as a fairly general guide for the determination of the loads for ordinary highway bridges.

"(a) For city bridges, subject to heavy loads;

"For the floor and its supports, a uniform load of 150 pounds per square foot of surface of the roadway and sidewalks, or a concentrated load of 20 tons on two axles 10 feet apart. In computing the floor beams and stringers the uniform load shall be assumed, together with either (1), this uniform load extending up to within ten feet of the wagon, or (2) the above specified concentrated load alone.

"For the trusses or girders, 100 pounds per square foot of floor surface for spans of 100 feet or less, 80 pounds for spans of 200 feet or over, and proportionally for intermediate spans. This uniform load is to be taken as covering the floor up to within ten feet of the wagon.

"(b) For suburban or town bridges, or heavy country highway bridges:-

"For the floor and its supports, a uniform load of 100 pounds per square foot, or a concentrated load of 15 tons on two axles 10 feet apart; these loads to be used as described under (a).

"For the trusses or girders, 80 pounds per square foot of floor surface for spans of 100 feet or less, and 60 pounds for spans of 200 feet or more, and proportionally for intermediate spans; to be used as described under (a). (Note d.)

"(c) For light country highway bridges:-

"For the floor and its supports, a uniform load of 80 pounds per square foot; this load to be used as described under (a). (Note d.)

"For the trusses or girders, 60 pounds per square foot of floor surface for spans of 75 feet or less, and 50 pounds for spans of 200 feet or more, and proportionally for intermediate spans; to be used as described under (a).

"(d) All parts of the floor of a highway bridge should be proportioned to carry a road roller weighing 15 tons.

and having three wheels or rollers, the weight on the front roller being 6 tons, and the weight on each rear roller to be 4.5 tons. The width of the front rollers to be taken as 4 feet, and of each rear roller 20 inches; the distance apart of the two rear rollers to be 5 feet centre to centre, and the distance between front and rear rollers 11 feet centre to centre. In using these loads, if the stringers are not over 2 1/2 feet apart on centres, each load shall be considered distributed equally on two stringers; and the fibre stresses allowed shall be 50 per cent above those specified in paragraph 18." i. e. (16000 +50%)."

Snow and ice load must also be considered in computing the stresses in draw spans when open since such stresses may attain considerable importance. The magnitude of these loads in the vicinity of New York will probably not exceed 10 lbs. per square foot. For fixed span bridges snow need not be taken into account since the maximum wagon and other loads will not occur simultaneously with the snow load.

13. LIVE LOADS for BUILDINGS. The proper loads for buildings depends upon the purpose for which the building is to be used, and in the larger cities is generally prescribed by law.

The live loads which follow are the minimum live loads prescribed by the present New York City building laws and represent good practice;-

FLOORS

Dwelling house, Apartment house; or hotel	60	lbs. per sq. ft.
Office building - First floor	150	" " " "
" " Other floors	75	" " " "
School house	75	" " " "
Stable or carriage house	75	" " " "
Place of public assembly	90	" " " "
Ordinary stores or light manufacturing or light Storage	120	" " " "
Stores where heavy materials are kept, warehouses, and factories	150	" " " "

ROOFS

All roofs with pitch less than 20°, 50 lbs.per.sq.ft.of surface
" " " " more " 20°, 30 " " " " " horizontal projection of surface.

"For columns of dwellings, office buildings, stores, stables, and public buildings when over five stories in height a reduction of the live loads may be made as follows;-

"For roof and top floor use full live load; for each succeeding lower floor reduce live load by five per cent, until fifty per cent, of the live loads fixed by this section is reached, when such reduction or such reduced loads shall be used for all remaining floors."

A complete discussion of the subject of floor loads would be too lengthy for these notes and the student is referred for further information to the article by C. C. Schneider in the "Trans. Am. Soc. C. E. Vol. LIV, page 371 et seq. with the ensuing discussion.

14 Wind Load. Wind pressure is a subject upon which little exact information exists, although many experiments have been made and much study given to the subject by engineers and scientists. Among the unsettled questions are;-

The relation between pressure and velocity.
The variation of pressure with size and shape of exposed plane surfaces.
The direction and intensity of pressure upon non-vertical surfaces.
The intensity of pressure upon non-planer surfaces.
The total pressure upon a number of parallel bars or other members placed side by side.
The decrease of pressure upon leeward surfaces.
The lifting powers of the wind.

In comment upon the above it may be said that the pressure varies about as the square of the velocity and that the results given by different experimenters vary from

$$P = .005V^2 \quad \text{to} \quad P = .0032V^2$$

of which the latter value represents the result of unusually careful experiments by Stanton * upon the intensity of pressure on plates varying in size for 25 to 100 sq. ft. and is probably more nearly correct than the higher value. In these formulas

P = pressure in pounds per square foot,
V = velocity in miles per hour.

In the Stanton formula the values are reduced to correspond to a temperature of 60° F and an atmospheric pressure of 14.7 lbs. per sq. inch.

The variation of pressure with size and shape of exposed surface is important and is not well understood, although it is sure that the resultant pressure on a large surface may be taken as less per square foot than that on a small surface, since the maximum intensity of the wind is due to gusts of comparatively small cross section.

The pressure upon vertical plane surfaces may be taken as normal to the surface and equal in intensity to the assumed wind pressure. Upon surfaces which are not vertical, the pressure is usually considered as normal to the surface but

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* See "Minutes of Proceedings of the Institute of Civil Engineers." Vols. 156 and 171.

lower in intensity than upon vertical surfaces. The variation in pressure with respect to the slope is not well understood and a number of empirical formulas are in use among which may be noted the much used Duchemin formula.

$$Pn = P \frac{2 \sin i}{1 + \sin^2 i} \qquad 14a$$

and the Hutton formula $Pn = P \sin i^{(1.84 \cos i-1)}$ 14b
In which P = intensity of normal pressure upon the given surface.

p = " " " " " a vertical surface.
i = angle made by surface with the horizontal.

A theoretical formula may be deduced by making the assumption that the wind always blows in horizontal lines, and that if the pressure be resolved into normal and tangential components the tangential component may be neglected. Upon this basis the following formula may be derived using the nomenclature just given.

$$Pn = P \sin^2 i \qquad 14c$$

This formula may be demonstrated in the following manner;-

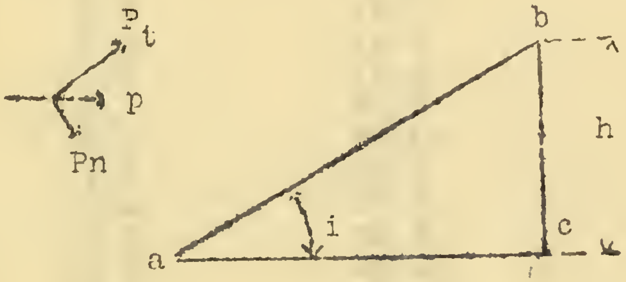


Fig. 14a.

Let the wind be assumed as blowing against the surface ab of height bc and making an angle i with the horizontal. Let the length of the surface be one foot, perpendicular to the plane of the paper.

Let P = intensity per sq. ft. of the horizontal wind force on a vertical surface.

Let Pn = intensity per sq. ft. of the normal force acting on surface ab.

Let Pt = " " " " " " tangential force acting on surface ab.

The total horizontal pressure on surface ab then equals Ph.

" normal component of this pressure = Ph sin i

" intensity of the normal component = $\frac{Ph \sin i}{ab}$

But $ab = \frac{h}{\sin i}$ $\therefore$ $Pn = P \sin^2 i$

This formula gives lower values than the empirical formulas 14a and 14b and probably gives too low values since it makes no allowance for the reduction in pressure on the leeward side which is known to exist and which may in part be attributed to the influence of the tangential component. It should also be noted that the wind does not blow uniformly in horizontal lines but may deviate considerably from this direction.

The values given by these three formulas are tabulated below for comparison using an assumed value of 30 lbs.per.sq.ft.for P.

i	$P \sin^2 I$	$P \frac{2 \sin i}{1 + \sin^2 i}$	$P \sin i \; (1.84 \cos i - 1)$
5°	0.0	5.2	3.9
10°	0.9	10.1	7.3
15°	2.0	14.6	10.5
20°	3.5	18.4	13.7
25°	5.3	21.5	16.9
30°	7.5	24.0	19.9
35°	9.9	25.8	22.6
40°	12.4	27.3	25.1
45°	15.0	28.3	27.0
50°	17.6	29.0	28.6
55°	20.1	29.4	29.7
60°	22.5		Above 60°
65°	24.6	Above 60°	use 30 lbs
70°	26.4	use 30 lbs.	
75°	28.0		
80°	29.1		
85°	29.7		
90°	30.0		

In the absence of further experience upon this phase of wind pressure it would seem wise to use one of the empirical formulas instead of the theoretical one, and the Hutton formulae, 14b, is used quite generally in England and the United States.

The following theorem relating to the wind pressure upon plane surfaces is particularly useful in determining reactions upon roof trusses;-

The horizontal component of the total normal pressure upon a plane surface equals the intensity of the normal pressure multiplied by the area of the vertical projection of the surface, and the vertical component of the total normal pressure equals that intensity multiplied by the area of the horizontal projection of the surface.

This theorem applies to any surface subjected to a uniformly distributed normal pressure and may be proven as follows;-

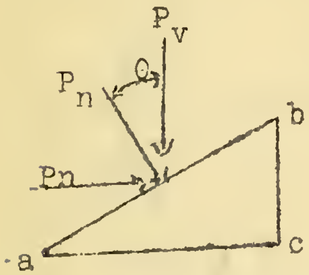


Fig. 14b.

Let pn = intensity of a normal force acting upon surface ab.
" P_H = horizontal component of total normal force upon ab.
" Pv = vertical component of total normal force upon ab.
" bc = vertical projection of ab.
" ac = horizontal " " "
" θ = angle between ab and horizontal.

Assume surface ab to be of length unity perpendicular to paper.

Then total normal pressure on ab = Pn x ab.

$$P_H = Pn \times ab \times \sin\theta = Pn \times ab \times \frac{bc}{ab} = Pn \times bc.$$

$$\text{and } Pv = Pn \times ab \times \cos\theta = Pn \times ab \times \frac{ac}{ab} = Pn \times ac,$$

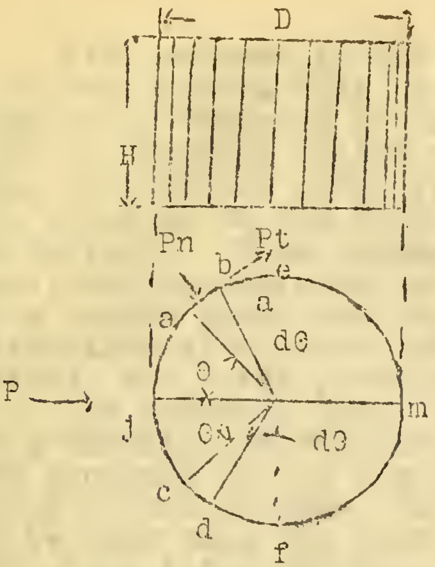
and since bc and ac are the vertical and horizontal projections of the area ab the theorem is proven.

It will be observed that if θ is greater than 45° P_H will be larger than Pv, and if less than 45° Pv will be greater than P_H. It is obvious that this is correct since the steeper the roof the greater the horizontal component. When θ = 90°, Pv = 0 and when θ = o, Ph = o.

The application of this method, to the solution of a problem is given in Article 25, problem 25c.

The pressure upon non-planar surfaces is important in the case of chimneys, stand-pipes, and other similar objects.

If the assumptions that were made in the deduction of formula 14c be also made for curved surfaces the total pressure upon each surface can be easily figured. The following demonstration shows the solution for the case of a vertical cylinder.



Let P = intensity of pressure on a vertical plane.
Let Pn = pressure on a plane making an angle of 90°−θ with the horizontal = P $\sin^2(90°-\theta)$

Let Pt = tangential component of pressure on same plane.

The normal pressure on the differential area ab subtended by the ∠ dθ = $P\sin^2(90°-\theta)\,\frac{HD}{2}\,d\theta$

As the tangential component Pt is neglected by hypothesis and the component of Pn acting upon surface ab in a direction parallel to ef is balanced by an equal and opposite component upon cd, the force tending to overturn the cylinder is the summation of the components of Pn parallel to gm.

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} p\sin^2(90°-\theta)\,H\,\frac{D}{2}\,d\theta\,\cos\theta$$

$$= \frac{PHD}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\cos^3\theta\,d\theta = \frac{PHD}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta\,(1-\sin^2\theta)\,d\theta$$

$$= \frac{PHD}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\sin\theta\,d\theta - \sin\frac{3\theta}{3}\,d\theta\right)$$

$= \frac{PHD}{2}\left(\frac{4}{3}\right) = \frac{2}{3}PHD = \frac{2}{3}$ the total pressure on a plane diametrical section.

In a similar manner the pressure on a spherical or conical surface may be computed.

The pressures obtained by this method lack experimental proof but are probably more nearly correct than the pressure obtained by the same method upon plane surfaces. The value given for the cylinder is quite generally used.

With respect to the total pressure upon a number of parallel bars placed side by side it may be estated that experiments previously referred to indicate that the total pressure on a pair of circular plates placed 1 1/2 diameters apart is less than that on one plate from which the conclusion is drawn, that the pressure on the leeward plate is in a direction opposite to current. When plates were placed 2.15 diameters apart the resultant pressure on the two plates was found to equal that on a single plate and the shielding effect was found to be well maintained with wider spacing, since at a distance of five diameters, the total pressure was only 1.78 that on a single plate.

The pressure upon the windward side of an exposed surface is a function of the density and velocity of the air currents. The pressure on the leeward side is also a function of the shape of the surface, and has been shown by numerous experiments to be less than the static pressure of the air current. The resultant total pressure upon a surface is in consequence not only a function of the direct pressure on the windward side but also of the pressure on the leeward side which in turn is a function of the form of the surface. It is therefore doubtful if an algebraical formula can be deduced which will give the pressure on surfaces of varying shape with any considerable degree of precision.

In the case of a very rapid reduction of atmospheric pressure as in a tornado it is often observed that building roofs are lifted and walls blown outward. This phenomenon is due to the air in the building which is under more or less restraint changing pressure less rapidly than the outside air and thereby producing a difference in pressure. This lifting action doubtless occurs to a greater or less degree whenever the external pressure is reduced, and should be guarded against by anchoring roofs securely to the walls.

The many uncertainties connected with wind pressure make worthless the attempts to specify with precision its magnitude and direction. In the lack of additional information and further theoretical studies there seems to be no reason from deviating from the common rules which have been used in bridge design for many years with satisfactory results. These rules as applied to bridge engineering may be stated as follows;-

The portal, vertical, and horizontal bracing shall be proportioned for a wind pressure of 30 pounds per square foot on the surface of a train averaging 10 square feet per linear foot and beginning 2 feet, 6 inches above base of rail, and on the exposed surfaces of the floor system and both trusses. The train pressure shall be considered as a moving live load, and the other pressure as a dead load. For trusses of ordinary spans the wind stresses shall also be computed upon the unloaded bridge for a pressure of 50 pounds per square foot, In the design the maximum stress computed by either of the above methods shall be used.

The wind stresses in the main truss members shall also be computed, but if the combined stress in any members due to dead load, vertical live load, and wind load does not exceed by more than 20% the allowable unit stress no allowance in the main members need be made for the wind.

In computing the area of exposed surface take twice the front surface of members composed of many bars, and 1.5 that of bars in pairs. The pressure upon the ends of ties, and upon the guard timbers should not be neglected and may be considered as one square foot per linear foot of bridge.

For wind pressure on roofs and buildings it is common practice to allow 30 pounds per square foot acting horizontally in any direction upon the sides and ends of buildings, or on the vertical projection of roofs. It is also very important to figure the wind stresses on the steel frame considering it as an independent structure without walls, floors, or partitions, since often failures occur in erection.

15 SNOW LOAD. The weight per foot of snow and ice varies greatly with climatic conditions. The following rule suggested by C.C. Schneider in the paper recently referred to gives reasonable results for conditions similar to those existing in Boston and New York.

"Use for all slopes up to 20° with the horizontal 25 lbs. per square foot of horizontal projection of roof. Reduce this value by one lb. for each additional degree of slope up to 45° above which no snow need be considered."

To determine the maximum stresses in a truss member wind and snow must be properly combined. The following combination may exist and should be considered.

Dead load with snow on both sides.
" " " " " one side and wind on the other.
" " " ice of 10 lbs. per square foot, properly reduced according to slope, on both sides, and wind on one side.

The maximum stress as determined by either of these combinations should be used.

For roof trusses of short span it is becoming the custom to combine the snow, wind, and dead load by using a value sufficient to cover them all. The following are suggested by Schneider as minimum loads per square foot of exposed surface to provide for combined dead, wind and snow loads on spans less than 80 feet. These loads are to be taken as acting vertically.

		Lbs
	(On boards, flat slopes, 1 to 6 or less	50
Gravel or composition	(" " steep " more than 1 to 6	45
	(" 3" flat tile or cinder concrete	60
Corrugated sheeting on boards and purlins		40
Slate on boards and purlins		50
" " 3"-flat tile or cinder concrete		65
Tile on steel purlins		55

Where no snow is likely to occur these values are to be reduced by 10 lbs., but no roof is to be designed for less than 40 lbs.

For roofs with other coverings than those above use 30 lbs. per sq. ft. of horizontal projection for combined effect of snow and wind on all slopes.

16 Centrifugal Force and Friction. For railroad bridges on curves the effect of centrifugal force must be considered. This may be computed by the following formula;-

$C = 0.03\ W\ D$ for a curvature up to 5 degrees where
C = centrifugal force in pounds.
W = weight of train in pounds.
D = degree of curvature.

The coefficient for centrifugal force (0.03) shall be reduced 0.001 for every degree of curvature above 5 degrees.

On trestle towers and similar structures the longitudinal thrust of the train must be considered. This may be taken as having its maximum value when the brakes are set and the wheels sliding, and may be computed assuming a value of 0.2 for the coefficient of sliding friction.

17 Impact on Railroad Bridges. It is easy to determine the weight per wheel applied to a railroad bridge by the locomotive or cars of a given train when at rest, but when in motion the effect of vibration, roughness of track, rapidity of application, and counterbalancing of locomotive driving wheels can not be determined theoretically and has not yet been precisely determined by experiment. Neither is the distribution of these loads by rails and ties a matter which can be easily ascertained. In consequence the engineer is compelled either to use a low unit stress, or to increase the live stresses by an allowance for "Impact" sufficient to cover these uncertainties. The latter method is more scientific and is coming into general use. Impact is used in Mechanics to mean the dynamic effect of a suddenly applied load, but as used in bridge engineering it stands for the increased stress produced in a member not only by the rapid application of the load but also by the other causes just mentioned and the term "coefficient of impact" is given to the factor by which the live stress must be multiplied to obtain the impact. No rational formula for determining this coefficient of impact has yet been deduced, but several empirical formulas are in more or less common use.

It is proven in mechanics that a load when instantaneously applied to a bar produced a stress exactly double that caused by the same load when gradually applied. In the ordinary structure the maximum load is, however, never applied instantaneously, though in short railroad bridges the length of time required to produce maximum moment or shear is very small. In consequence sudden application alone is never sufficient to double the live stresses as computed for quiescent loads. Many engineers, however, use for short spans a coefficient equal to unity assuming that the effect of vibration and other uncertainties is balanced by the difference between the stress due to instantaneous application, and that due to the very rapid but not instantaneous application caused by a railroad train. For longer spans the coefficient is generally reduced.

[illegible]

[illegible]

W = weight of train in pounds.

[illegible]

[illegible] coefficient of sliding friction.

Impact on Railroad Bridges. It is easy to [illegible]

[illegible]

The two following formulas show two different types of impact formulas;

From American Bridge Co, Specifications $I = S \dfrac{300}{L + 300}$

From Specifications of the Department of Railroads and Canals, Canada,

$$I = S \frac{S}{S + D}$$

In above formulas

L = length in feet of distance which must be loaded to produce maximum live stress in a member.

S = that maximum live stress, and
D = the dead stress.

It will be seen at once that for short spans the coefficient in the first formula is very nearly unity, and that this also applies to the second formula unless the bridge is unusually heavy or the live loads very light. The first formula has been quite generally adopted by American engineers and while purely empirical agrees reasoably well with such experiments as have been made, and is in a logical form. It will be used hereafter in this book.

18 Impact for Highway Bridges and Buildings. Allowance for impact upon these structures may usually be less than for railroad bridges. On highway bridges of moderate length the practice of allowing 25% for import is not unusual, this being intended to cover the jolting effect of wagons on rough pavement, and the impact of electric cars. It should be noted that the loads on highway bridges are probably seldom or never applied with sufficient speed to make rapidity of application an important element, and that the impact of electric cars is not as severe as that of locomotives since the motion of the machinery is rotary rather than reciprocating. For long span highway bridge trusses it is usual to make no allowance for impact.

For buildings it is customary to make no allowance for impact except where moving cranes or other shock producing machinery are used.

Before leaving the subject of impact it should be noted that it is probable that the effect of impact upon wooden beams is less injurious than upon steel beams owing to the greater elasticity of the wood, and that some engineers disregard impact in designing wooden structures.

19 Inner Forces, The allowable inner forces depend upon the character of the material and loading, the precision with which the stress can be computed, and the uses to which the structure is to be put. The best modern practice is based upon the reduction to an equivalent dead load by the addition of impact of the stresses due to the live loads and if this can be done the character of the loading may be neglected. In what follows

19 The allowable working unit stresses for a given structure depends upon the material, the character of loading, the precision with which the stresses can be computed, and the uses to which the structure is to be put. If proper allowance for impact be made the character of loading may, however, be neglected.

The following unit stresses represent good practice for ordinary structural steel. Structures provided proper allowance for impact be made.

WORKING STRESSES. STRUCTURAL STEEL.

Live Load increased to allow for Impact.

Tension, direct compression, and extreme fibre stress in bending	16000	lbs. per. sq. in.
Compression in columns	$16000 - 70\frac{L}{R}$	" " "
Shear on net section of plate girder webs and on shop rivets.	12000	" " " "
Bending on pins	24000	" " " "
Bearing on pins and shop driven rivets.	24000	" " " "
Bearing on hand driven rivets	18000	" " " "
Shear " " " "	9000	" " " "
Modulus of elasticity	28,000,000	

$\underline{L}$ = maximum of value ratio of unsupported length of column to radius of gyration, both values being expressed in inches.

The following values with the exception of tension are recommended for timber for railroad bridges by the American Railway Engineering and Maintenance of Way Association. These values may be used for green timber and without allowance for impact. For highway bridges and trestles these figures may be increased by 25%, and for buildings when protected from weather and reasonably free from impact by 50%. For values for other timber than yellow pine see "Proceedings of the American Railway Engineering and Maintenance of Way Association" Vol. 10-Part I.

WORKING STRESSES. LONG-LEAF YELLOW PINE.

No allowance for Impact required.

Bearing along grain	1300	lbs.per.sq.in.
Compression in columns, length over 15 diameters	$1300\left(1-\frac{L}{60D}\right)$	" "
Tension parallel to grain	1400	lbs.per.sq.in.
Bending---Extreme fibre stress	1300	" " " "
Shearing along grain in beams	120	" " " "
" " " " chord blocks, etc.	180	" " " "
Bearing across grain	260	" " " "
Modulus of elasticity	1,600,000	

$\frac{L}{p}$ = maximum value of ratio of unsupported length of column to radius of gyration both values being expressed in inches.

For deflection of yellow pine beams under long-continued loading, use for E. 800,000.

The following values for bearing on masonry represent good practice.

WORKING STRESSES. BEARING ON MASONRY.

Live load increased to allow for Impact.

Granite Masonry and Portland Cement concrete	600 lbs. per. sq. in.
Sandstone and limestone	400 " " " "

20 Factor of Safety. The unit stresses given in the previous article are all much less than the breaking strength of the material, the ratio between the breaking strength and the allowable unit stresses being known as the "factor of safety." The necessity for using such a low value is due to the following facts.

1st. Material can not be stressed with safety above the elastic limit which is generally not more than one half the breaking strength.

2nd. The magnitude, point of application, and distribution of the live loads as well as the allowance for impact is approximate.

3rd. The material used is variable in quality when rolled, and may be injured.in fabrication.

4th. The effect of changing conditions can not be predicted. This applies to character and amount of loading and to deterioration of material through rust, or rot.

5th. The common theories give primary stresses only and neglect the secondary stresses due to distortion of the structure, these additional stresses being sometimes of considerable importance.

CHAPTER II.

LAWS OF STATICS-REACTIONS, SHEARS AND MOMENTS, INFLUENCE LINES.

1. Laws of Statics. The theory of structures is based upon the fundamental principles of statics and these the student must thoroughly understand. They may be stated as follows;

A system of coplanar forces is in equilibrium if the following conditions are satisfied;

1st. The algebraic sum of the components of all the forces acting parallel to any axis in the plane of the forces must equal zero.

2nd. The algebraic sum of the moments of all the forces about any axis at right angles to the plane of the forces must equal zero.

If the forces be resolved into components parallel to two rectangular axes, OX and OY, and the algebraic sum of the forces parallel to OX be designated as ΣX and of these parallel to OY as ΣY the first of above conditions will be fulfilled when $\Sigma X = 0$ and $\Sigma Y = 0$, hence the two principles stated above are fully comprehended by the three following equations.

$$\Sigma X = 0 \qquad \Sigma Y = 0 \qquad \Sigma M = 0$$

If the forces acting upon a body do not satisfy all of these three equations then the body cannot be in equilibrium. For example if $\Sigma X = 0$ and $\Sigma Y = 0$, but ΣM does not, then the body is in a condition of rotation about a stationary axis. If $\Sigma Y = 0$ and $\Sigma M = 0$, but ΣX does not, then the body has a motion of translation in a direction parallel to the X axis but no other motion.

In practice it is common to use horizontal and vertical axes for which case the first equation may be written $\Sigma H = 0$ and $\Sigma V = 0$.

22 Reactions. Each of the reactions upon a structure may have three unknown properties, viz; magnitude, direction, and point of application. Usually, however, the point of application of each reaction, is fixed in position and the direction of at least one of the reactions is known. If this condition exists when there are two points of support, i. e., two reactions, as

is the case in most structures; there remain but three unknown properties of the reactions, all of which may be computed by the three equations of statics, and the structure is statically determined with respect to the outer forces, whether it is or is not possible to determine the inner stresses by statics. If there are more than three unknown properties of the reactions, e. g. if only the points of application are fixed; or if the structure is supported on more than two points, then it is statically undetermined with respect to the outer forces, unless some special form of construction is adopted as in the three-hinged arches and cantilever bridges considered later. If there are fewer than three unknowns then the structure is in general, unstable and will tend to move bodily under the applied loads unless these fulfil certain special conditions.

Illustrations of the above conditions is afforded by the structures shown in Fig. 22a, 22b, 22d, and 22d, for all of which the position and magnitude of the applied loads, and all of the dimensions of the structure are supposed to be known.

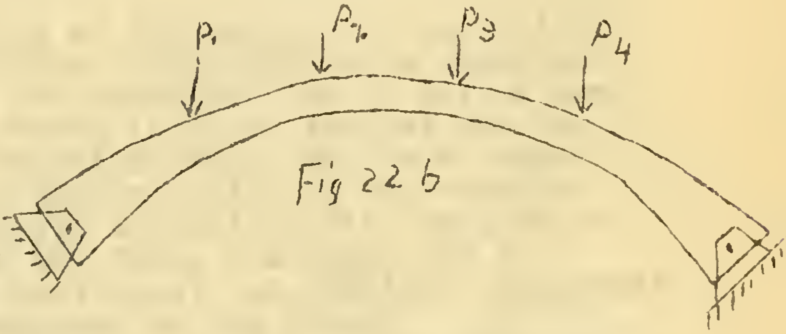


Fig 22 b

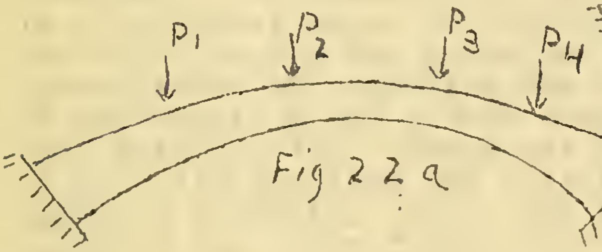


Fig 22 a

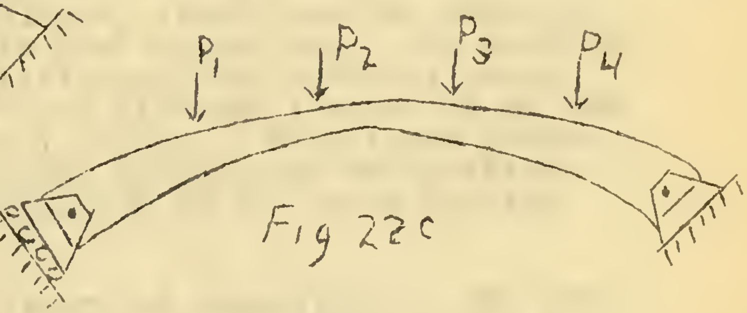


Fig 22c

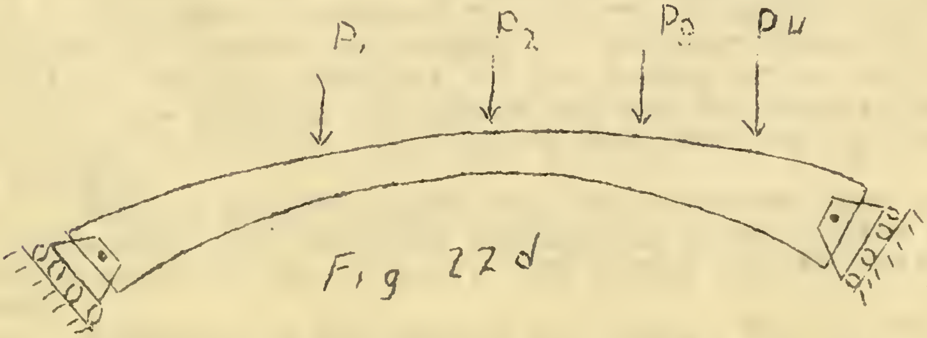


Fig 22d

is the case in most structures; there remain but three unknown components of the reactions, all of which may be computed by the three conditions of equilibrium, and the structure is statically determinate with respect to its outer forces, whether it is or is not possible to determine the loads required by statics. If there are more than three unknown properties of the reactions, e. g. if only the points of application are known, or if the structure is supported on more than two points, then it is statically indeterminate with respect to the outer forces, unless some special form of construction is adopted as in the three-hinged arches and cantilever bridges considered later. If there are fewer than three unknowns then the structure is in motion, unstable and will tend to move bodily under the applied loads unless these fulfill certain special conditions.

Illustrations of the above conditions is afforded by the structures shown in Fig. 21a, 21b, 21c, and 21d, for all of which we consider the magnitude of the applied loads, and all of the dimensions of the structure are supposed to be known.

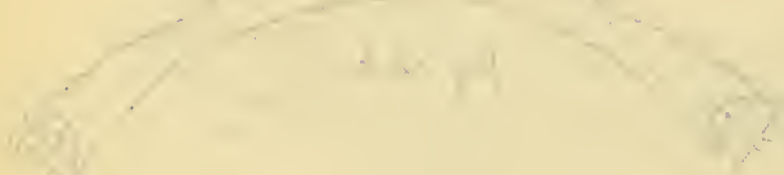


Fig 21a

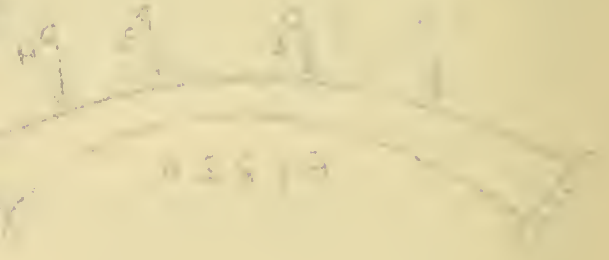


Fig 21b

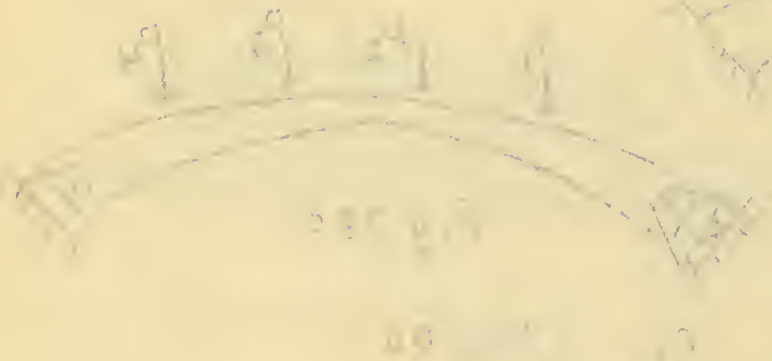


Fig 21c

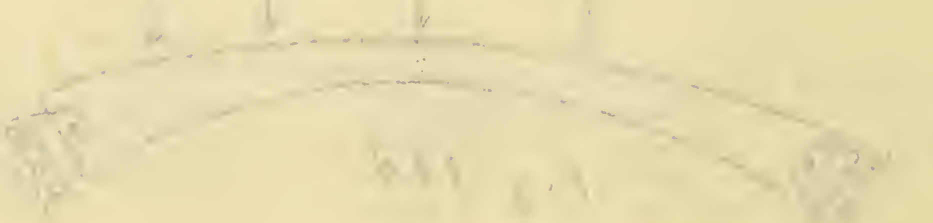


Fig 21d

Fig. 22a. represents the ordinary masonry arch in which each reaction is unknown in direction, magnitude, and point of application. In consequence the structure is indeterminate with respect to the outer forces in a three-fold degree. Fig. 22b shows a two-hinged arch which has the point of application of both reactions determined but the magnitude and direction of neither are known, hence it is indeterminate in the first degree with respect to the outer forces. In Fig. 22c, a set of rollers is shown at one end. The function of these rollers is to make the reaction at that end perpendicular to the supporting surface since the rollers, if in good condition can offer but little resistance to motion along this surface. This structure is, therefore, statically determined with respect to the outer forces since the points of application of both reactions and the direction of one are known. In Fig. 22d rollers are shown at both ends hence the direction of both reactions are known. Unless these reactions meet on the line of action of the resultant of the applied loads equilibrium can not exist and the structure will move, therefore this structure is unstable.

23 Computation of Reactions--Method of Procedure. It is evident that if the horizontal and vertical components of a reaction which is unknown in direction and magnitude, or if either component of a reaction which is known in direction but not in magnitude, be determined the reaction itself may be at once obtained. In consequence the determination of the reaction in a structure which is statically determined with respect to the outer forces and hence has but three unknowns, may be accomplished by computing the horizontal and vertical components of one reaction and either component of the other.
This method often, though not always, simplifies the solution of reaction problems and will be used hereafter. Its adoption makes it desirable to use the horizontal and vertical components of the outer forces and these also can frequently be computed more easily than the actual forces. With these components of the outer forces known the solution of the problem may be accomplished by the application of the three statical equations.

The following mode of procedure is suggested for the use of the beginner who is advised to follow it exactly until he has mastered the method thoroughly. For structures in which the reactions are not parallel to the forces or in which the character of the unknown reactions can not be easily predicted, even the experienced computor should not omit any of the steps in the process.

1st. Draw a careful sketch of the structure and show on it the horizontal and vertical components of the outer forces. This sketch need not be to scale but should not be materially distorted.

2nd. Indicate on the sketch by arrows, and by the letters H and V the assumed components of the reactions, using letters R and L as suffixes of H and V to indicate right and left reactions.

The direction of the components of the reactions which are unknown in direction may be assumed at random, e, g., the horizontal component may be assumed as acting either to the right or the left and the vertical component either up or down, but the components of the reaction the direction of which is known must be assumed so that they will be consistent with this known direction.

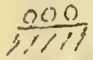
3rd. Determine the unknown H and V components by the solutions of the equations $\Sigma H = 0$, $\Sigma V = 0$, and $\Sigma M = 0$, considering as positive forces acting upwards or to the right, and clockwise moments.

A positive result for a component shows that the component in question acts in the direction originally assumed, and not necessarily that it acts up or to the right. With the magnitude of all components known the magnitude of either reaction may be obtained by computing the square root of the sums of the square of its two components. Its direction is determined by the direction of the components. The beginner is more likely to make errors by omitting some of the forces than in any other way. Particular attention may well be called to the fact that horizontal forces may produce vertical reactions and vice versa.

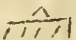
If the load, or any portion of it, be distributed over a considerable distance instead of being applied at a point, the resultant of this portion of the load may be used in the computations as a concentrated load. This method, however, should be used only in reaction computation; it would in general be incorrect for the determination of shears, moments, and truss stresses as will be shown later.

It is always desirable to obtain a check by twice applying the equation, $\Sigma M = 0$, once about each point of support. This gives an independent check for at least one of the reaction components, which in the case of a simple beam with vertical loads is sufficient and conclusive.

24 REACTION CONVENTIONS. Hereafter, in both text and problems, structures supported at one end upon a set of rollers or by a tie-rod will be considered as having the reaction at that point fixed in direction. The reasons for this in the case of rollers as stated in A rticle 22. For the case with the tie-rod, it is sufficient to recall that such a rod is little better than a stiff rope and is incapable of carrying bending or compression, hence the reaction which it carries must act along its axis and produce tension in the rod.



Rollers will be indicated by this conventional symbol, and the reaction in this case is always to be considered as perpendicular to the supporting surface, whether the surface be horizontal, inclined, or vertical.

When the point of application of a reaction is fixed but not its direction this symbol  will be used. This is not intended to represent a knife edge bearing since the reaction may act in any direction; i. e. up, down, horizontal or inclined. If this symbol be combined with rollers, then both point of application and direction of reaction are to be considered as fixed. If the reaction is carried by a tie rod, the rod will be so marked; in this case the point of applica-

tion should be taken at the point where the rod is fastened to the structure.

25 POINT OF APPLICATION OF LOADS AND REACTIONS. In practice it is seldom that the point of application of load or reaction is definitely fixed; it is however in many cases fixed within such small limits that no error arises in considering it as located at a definite point. This is the case when the structure is supported on steel pins as in most bridges of considerable size; the reaction in such a case passes through the pin which is generally but a few inches in diameter and its resultant will pass through the pin centre, or nearly so, unless the pin be badly turned or the bearing surface upon which it rests imperfect. With wheel loads the load acts at the point of tengency of wheel and bearing surface which is practically a point, but as the wheel does not rest directly on the structure but has its load distributed by rails and ties, or by the floor if a highway bridge, it is not applied to the structure itself at a point, though it is generally so considered as the error thus arising is small and on the safe side.

For ordinary beams which rest at the ends upon steel bearing plates inserted to distribute the load over the masonry supports the assumption that the reaction is applied at the centre of bearing is by no means an exact one. The actual distribution of the reaction in such a case is a function of the relative elasticity of the beam and support. If both beam and support were to be absolutely rigid, an impossible case, the reaction would pass through the centre of bearing; if the support alone were to be rigid the reaction would pass through the edge of the bearing plate; in the actual case where both beam and bearing surface yield to some extent the reaction is distributed over the entire surface and its intensity varies uniformly or nearly so, as shown in Fig. 25c. It will be noticed that the resultant pressure acts at a point between the centre of bearing and inner edge of the masonry. The common assumption for such cases is to assume the reaction as applied at the centre of bearing. This assumption is on the safe side in designing the beam as a whole, but on the unsafe side in proportioning the area of bearing. However, the error for short beams which deflect but little, is not serious. For long girders which deflect considerably the end bearing is usually made by a pin which is supported upon a shoe which in turn rests upon rollers, thus ensuring a uniform distribution of the reaction.

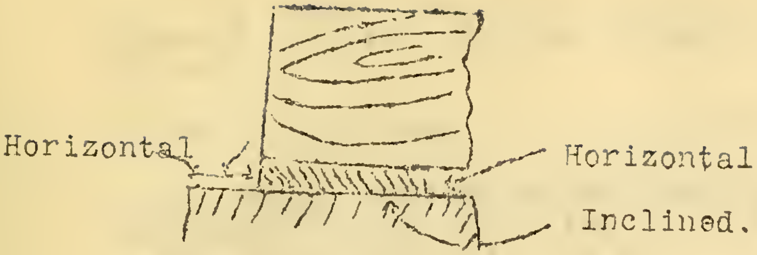


Fig. 25a.

Exaggerated detail of end of a loaded beam. Notice that masonry as well as beam are shown as deflected.

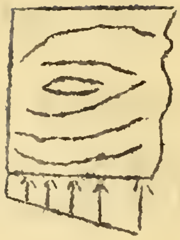


Fig. 25b.

A reasonable assumption as to the distribution of pressure at the left end of beam shown in Fig. 25a.

26 SOLUTION OF REACTION PROBLEMS. The application of the methods of Article 23 is illustrated in problems 26a, 26b, 26c and 26d of this article.

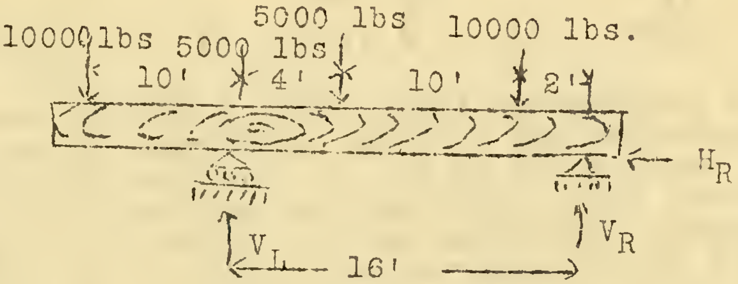


Fig. 26a.

Prob. 26a. Compute horizontal and vertical components of reactions on this beam due to loads shown. Neglect weight of beam itself.

SOLUTION. Apply first $\Sigma H = 0$. This gives the equation $H_R = 0$ since the applied loads are all vertical and in consequence have no horizontal components.

Now apply $\Sigma M = 0$ taking for an origin of moments the point of application of either reaction thus eliminating one unknown. The equation which follows is derived by taking moments about the right end

$$-10000 \times 26 + 16V_L - 5000 \times 12 - 10000 \times 2 = 0.$$

$$\therefore 16V_L = 340000 \text{ ft. lbs. and } V_L = +21250 \text{ lbs.}$$

Since the value of V_L is positive the reaction must act in the direction assumed on the sketch, i. e. upward.

The application of $\Sigma V = 0$, using the value of V_L just obtained gives the value of V_R and completes the solution of the problem. The equation follows:

$-10000 + 21250 - 5000 - 10000 + V_R = 0 \quad V_R = +3750$ lbs.

and [illegible] acts upward as shown.

To check this value apply $\Sigma M = 0$ using the left point of support for the origin of moments. The expression thus obtained is

$$-10000 \times 10 + 5000 \times 4 + 10000 \times 14 - 16 V_R = 0 \therefore V_R = +3750 \text{ lbs.}$$

which checks the value obtained by the application of $\Sigma V = 0$ and hence checks the value of V_L since this was used in the original determination of V_R.

Prob. 26b. Compute horizontal and vertical components of reactions on this beam, neglecting weight of beam.

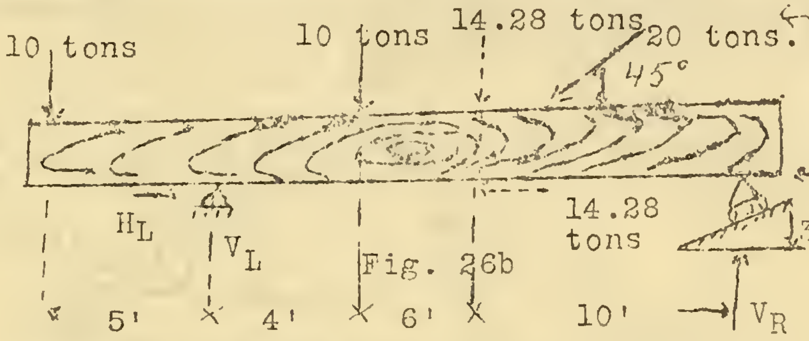


Fig. 26b

This force is replaced by its hor.and vert. components and is not used itsel

In this problem V_L and H_L are independent of each other in magnitude and direction and each may be assumed as acting in either direction. V_R and H_R are however mutually related both in direction and magnitude since their resultant must act at right angles to the supporting surface and hence make an angle of 60° with the horizontal. To fulfil this condition if V_R is assumed as upward H_R must be assumed to the left. The ratios of their magnitude equals the ratio of the sides of a 60° triangle as indicated by Fig. 26c. hence $V_R = H_R \cot 30° = 1.71 H_R$.

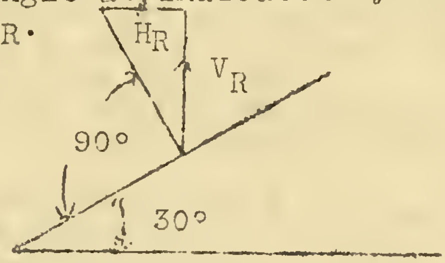


Fig. 26c.

To solve this problem apply the equation $\Sigma M = 0$ taking moments about the point of application of the right hand reaction. The following equation results

$$-10 \times 25 + 20V_L - 10 \times 16 - 14.28 \times 10 = 0$$

which when solved gives $V_L = +27.64$ tons ΣV_L acts upward as assumed.

It will be noticed that the lever arms about the origin of moments of all the horizontal forces are zero hence these terms do not appear in the equation. Had the inclined force been resolved at the top instead of the bottom of the beam this condition would not have existed but the value of the reaction would not have been changed since the moment of the horizontal component would have been neutralized by the change in the moment of the vertical component due to its altered

lever arm. *

The equations $\Sigma V = 0$ may now be used. This gives the following expression;-

$-10 + 27.64 - 10 - 14.28 + V_R = 0$ hence $V_R = +6.64$ tons and acts as shown.

Since $H_R = 1.73 V_R$ its value is found to be 11.49 tons.

The application of $\Sigma H = 0$ completes the solution by giving the value of H_L. The equation is $H_L - 14.28 - 11.49 = 0$ hence $H_L = 25.77$ tons and acts as shown.

To check the value of V_R take moments about the left point of support. This gives the following expression;
$-10 \times 5 + 10 \times 4 + 14.28 \times 10 - 20 V_R = 0$ whence $V_R = +6.64$ tons, thus checking the value previously obtained and in consequence the value of V_L.

As an independent check of H_R and H_L cannot readily be made a second computation of their value should be carried through, or the original computations carefully revised, the former being the safest method.

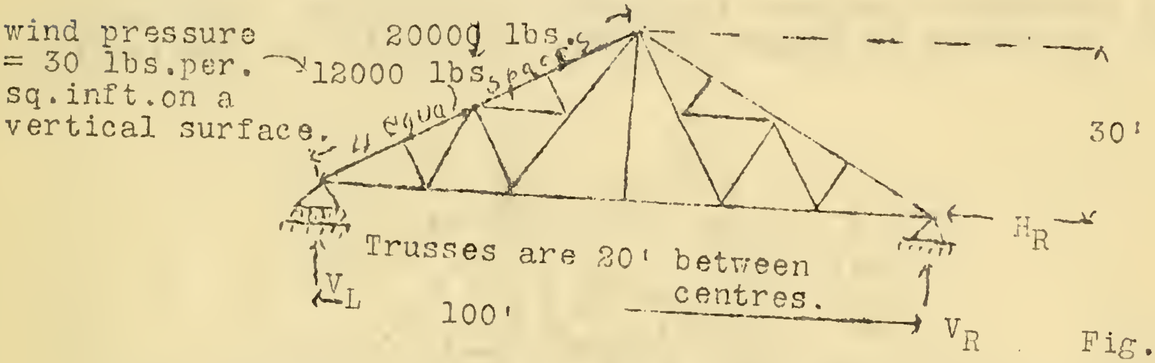

wind pressure = 30 lbs.per. sq.inft.on a vertical surface.
20000 lbs.
12000 lbs.
4 equal spaces
30'
H_R
Trusses are 20' between centres.
V_L
100'
V_R

Fig.25c.

Since the slope of the roof surface in this problem is about 30° it will be assumed that the normal intensity of the wind pressure is 20 lbs.per sq.ft. (See table in Article 14, Hutton's formula). The roof trusses are 20 ft.between centres hence the area upon the windward side of the building which is supported by one truss has a length of 20 ft. for intermediate trusses, and 10 ft. for end trusses. The reactions upon an intermediate truss will be computed.

Using the method of Article 14 the horizontal and vertical components of the total wind pressure on the windward side are found to be;-
PH = intensity of normal pressure x vertical projection of surface = $20 \times 30 \times 20 = 12000$ lbs.
PV = intensity of normal pressure x horizontal projection of surface = $20 \times 50 \times 20 = 20000$ lbs.

The truss may now be considered as loaded with the two forces of 20000 lbs. and 12000 lbs. acting at centre of windward surface, and the reactions due to these forces computed in the following way;
Applying $\Sigma M = 0$ about right end gives $100 V_L + 12000 \times 15 - 20000 \times 75$ whence $V_L = 13200$ lbs, acting up as assumed.
Applying $\Sigma H = 0$ gives $12000 - H_R = 0$,
whence $H_R = 12000$ lbs acting to left as assumed.

FOOT NOTE. The device of resolving a force into its components at a point where the lever arm of one of the components is zero is a very useful one, and frequently saves considerable labor. Its correctness is evident since the effect of a force upon a body as a whole always equals that of its components, no matter at what point the force is resolved nor what may be the direction, or length of the lever arms of the components, hence if the lever arm of one of the components is zero the moment of the force equals the moment of the other

Applying $\Sigma V = 0$ gives $13200 - 20000 + V_R = 0$
hence $V_R = +6800$ lbs acting up as assumed.

Applying $\Sigma M = 0$ about left end as a check gives $-100V_R + 20000.$
$x\ 25 + 12000\ x\ 15 = 0$
whence $V_R = +6800$ lbs. acting up as assumed,
and agreeing with value previously obtained.

Prob. 26d. Compute horizontal and vertical components of the reactions on this crane. Neglect weight of structure itself.

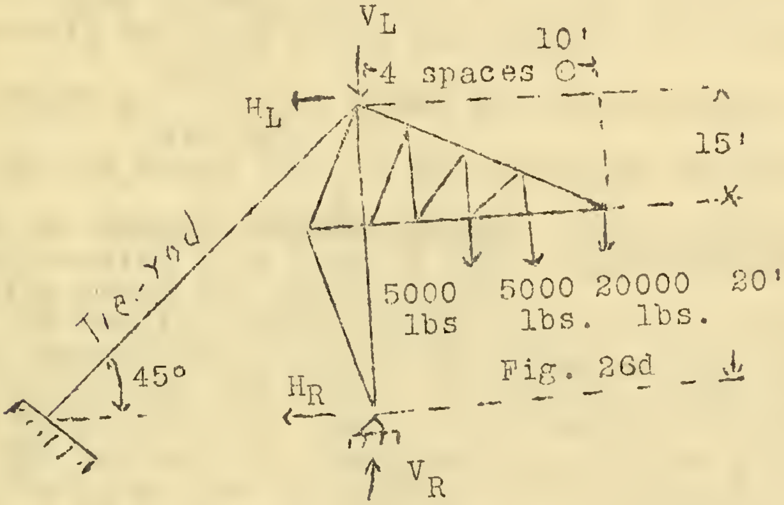


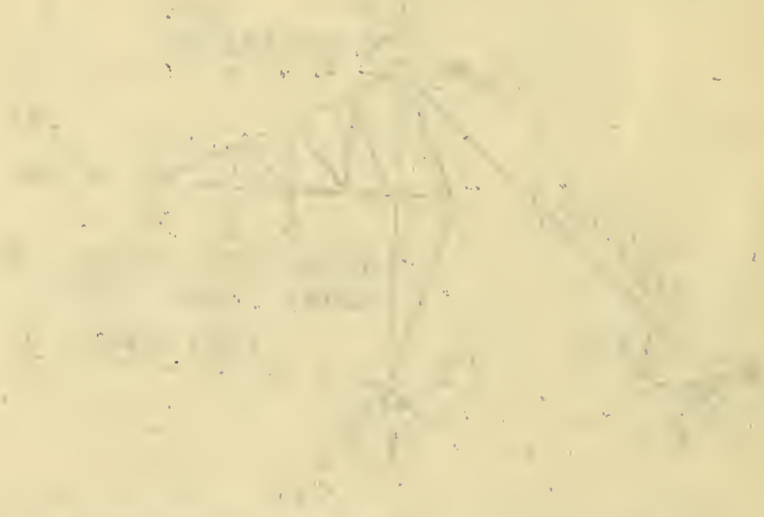
Fig. 26d

The direction of the reaction at top of the crane is fixed by the tie-rod, hence V_L and H_L can not be assumed to act at random but must be so chosen that their resultant will act along the tie-rod. The magnitude of their two components will, of course, be equal since the tie-rod makes an angle of 45° with the horizontal.

Applying $\Sigma M = 0$ about the bottom gives
$- 35H_L + 5000\ X\ (20+30) + 20000\ X\ 40 = 0$

hence $H_L = +30000$ lbs acting as assumed.

Since the two components of the tie-rod stress are equal $V_L = 30000$ lbs. also acting as assumed.



Applying $\Sigma H = 0$ using the value previously found for M_L gives
$30000 + H_R = 0$ hence $H_R = -30000$ lbs. acting to the
right and not as assumed.
Applying $\Sigma V = 0$ using the value previously found for V_L gives
$-30000 - 5000 - 5000 - 20000 + V_R = 0$
hence $V_R = +60000$ lbs acting up as assumed.
Applying $\Sigma M = 0$ about the top as a check for H_R and V_R
gives $+35\ H_R + 5000 \times 20 + 5000 \times 30 + 20000 \times 40 = 0$, hence $H_R = -30000$ lbs. checking the value
previously obtained.

It is always adviable to assume the reactions as acting in their probable directions to avoid complications. The opposite assumption was made for H_R in this problem in order to illustrate the solution with an incorrect assumption. The results will be found to agree in any case provided the work is correctly done, but it is confusing to have the reaction incorrectly indicated on the sketch. Sometimes, however, it is impossible to foretell the actual direction of a reaction.

In this problem the actual value of the reaction at the top should be found since this gives the tension in the tie-rod.

$$\text{This value} = \frac{30000}{\sin 45^\circ} = 42430 \text{ lbs approximately.}$$

This should equal $\sqrt{V_L^2 + H_L^2}$ which may be used as a check.

27 SHEAR and BENDING MOMENT DEFINED. Shearing force or shear, at any section of a body is that force which tends to produce slipping along the given section.

The bending moment at any section of a body due to a set of co-planar forces is the resultant moment about an axis passing through the centre of gravity of the section of all the forces on either side of the section it being understood that the section is perpendicular to the plane of the forces.

Fractures due to shear are due either to transverse fracture of the grains or fibre, or to their slipping upon one another.

In the ordinary structural materials wood is the only one of a fibrous character and shearing failures in this material ordinarily occur by longitudinal slipping of the fibre.

Fractures due to bending are caused by longitudinal failure of the fibres, either by tension or crushing.

28 METHOD of COMPUTATION, SHEAR and BENDING MOMENT. The magnitude of the shear upon a given section due to a set of co-planar forces may be readily computed as follows;- Resolve each force into two components parallel and perpendicular, respectively, to the given section. The algebraic sum of the components parallel to the section of all the forces upon either side of the section equals the shear. That either side of the section may be considered is evident from the fact that for structures in equilibrium $\Sigma Y = 0$ hence the algebraic sum of the forces on one side of the section and parallel to the Y axis must be equal in magnitude and opposite in direction to the corresponding term for the other side of the section.

Applying (2) and using the ratio previously found for M, gives [illegible]
[illegible]
[illegible] as assumed.

Applying (1) [illegible] using the value previously found for [illegible]
[illegible]
[illegible]

Applying (3) [illegible]
[illegible]
[illegible]

It is always desirable to check the resultants by making [illegible] independent [illegible] computations. [illegible]

This value [illegible] which is approximately [illegible]

7 SHEAR AND BENDING MOMENT DIAGRAMS. [illegible] at any section of a body [illegible] lying along the given section.

The bending moment at any section of a body due to a set of coplanar forces is the resultant moment [illegible] of all the forces on either side of the section [illegible]

[illegible]

In the [illegible]

[illegible] of the [illegible]

8 [illegible] SHEAR AND BENDING MOMENT. [illegible] of the forces upon a given section [illegible] parallel to the given section. [illegible] of the section [illegible] of the forces on one side of the section [illegible] the other side of the section.

The magnitude of the bending moment upon a given section of a set of co-planer forces may also be computed by resolving the forces into horizontal and vertical components. For this case, however, it is necessary to include the moments of both sets of components, though again it is immaterial which side of the section is considered in computing the moment.

29 SHEAR and BENDING MOMENT SIGNS. The signs for shear and bending moments must be used with care or errors will occur. Any reasonable convention may be adopted but it should be carefully observed that positive shear may represent forces acting in exactly opposite directions and that positive bending moment may stand for either clockwise or counter-clockwise moment, depending in both cases upon the side of the section considered in making the computation. The distinction between moment of forces in general as used, for example, in determining reactions, and moment upon a cross section of a beam should be carefully observed. In the former case clockwise moments should always be taken as of the same sign since the effect of such a moment upon the body as a whole is the same no matter upon what part of the body it may act. In the beam, however, clockwise moment upon the left of a given section produces the same effect upon the fibres as does counter-clockwise moment upon the right. In both cases compression is produced in the fibres of the upper portion of the section and tension in those of the lower portion.

30 SHEAR and MOMENT-COMMON CASES. In ordinary practice it is seldom necessary to compute shears or moments except for vertical sections of horizontal beams, and for trusses carrying VERTICAL loads. For such cases the following conventions may be adopted.

SHEAR. The shear upon a vertical section of a beam or truss equals the algebraic sum of all the outer forces (including reactions) upon either side of the section. It is positive when the resultant is UPWARD on the LEFT of the section or DOWNWARD on the RIGHT.

MOMENT. The moment upon a vertical section of a beam or truss equals the algebraic sum of the moments of all the outer forces (including reactions) upon either side of the section, about any horizontal axis lying in the section. It is positive when the moment of the forces on the left of the section is CLOCKWISE, or when the moment of the forces on the RIGHT of the section is COUNTER-CLOCKWISE.

31 CURVES of SHEAR and MOMENT DEFINED and DESCRIBED. A curve of shears or of moments is a curve the ordinate to which at any section represents the shear or moment at that section due to the applied loads. If the load be uniformly distributed the curve may be a continuous smooth curve, a series of smooth curves or a series of straight lines. If the loading consist of a series of concentrated loads the curve will always be composed of a series of straight lines. If the loading be a combination of concentrated and distributed loads the curves may be composed of a combination of straight and curved lines.

The magnitude of the bending moment [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

It should be thoroughly understood that these curves represent the effect of loads which are *fixed* in *magnitude* and *position*. The shear and moment due to a set of moving loads constantly varies and hence can not be represented by such curves except in certain definite cases such as that of maximum shear. The effect of moving loads is shown more clearly by influence lines which are explained later. Typical curves of shears and moments are shown in figures 31a to 31 f inclusive.

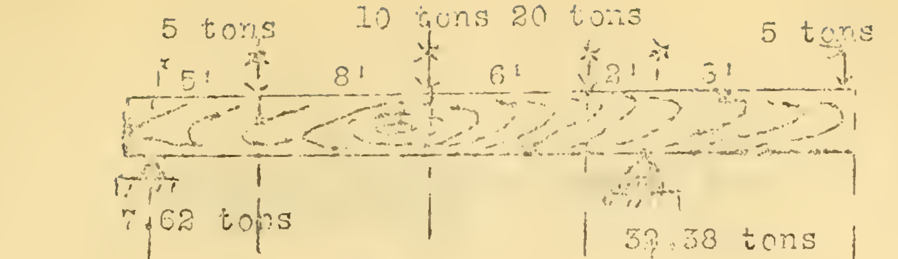


Fig. 31a.

In following curves weight of beam is neglected.

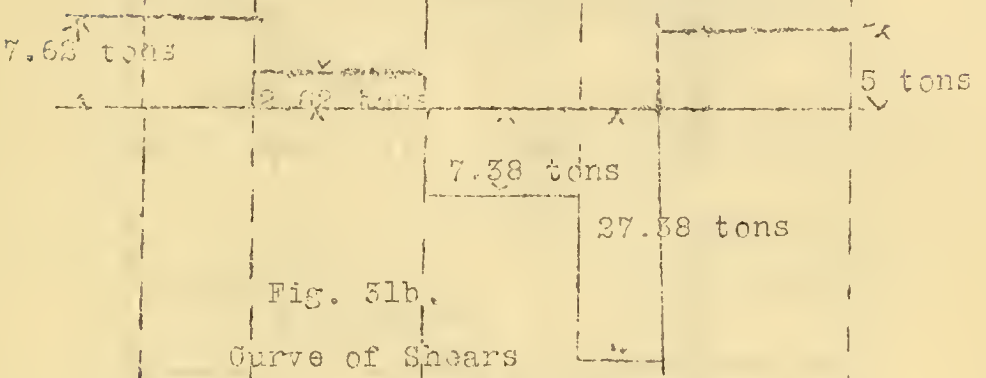


Fig. 31b.
Curve of Shears

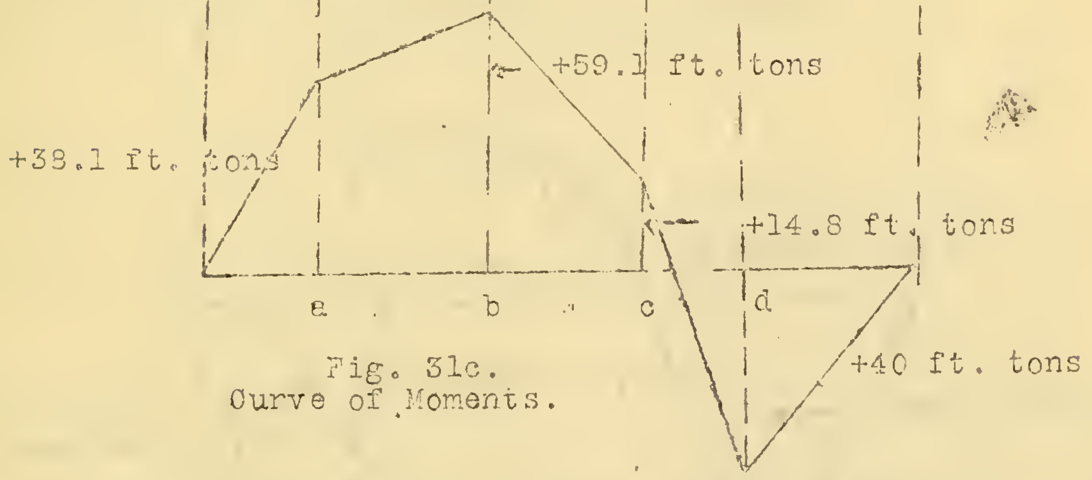


Fig. 31c.
Curve of Moments.

It should be noticed that the ordinate to the curve of shears at any section equals the algebraic sum of the forces acting on either side of the section, and that the curve of moments reaches both its maximum positive and maximum negative values at points where the curve of shear crosses the axis.

This latter relation always exists and is demonstrated in Article 33.

The value of the ordinate to the curve of moments at the points, a,b,c, and d are computed below

At a 7.62 x 5 = +38.10 ft. tons.
" b 7.62 x 13 - 5 x 8 = +59.06 " "
" c 7.62 x 19-5x14-10x6 = +14.78 " "
" d -5x8 = -40.0 " "

Curve of Shears and Moments for applied loads.

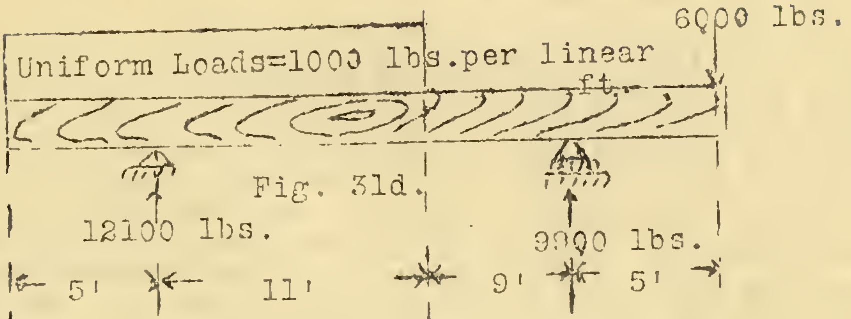


Fig. 31d.

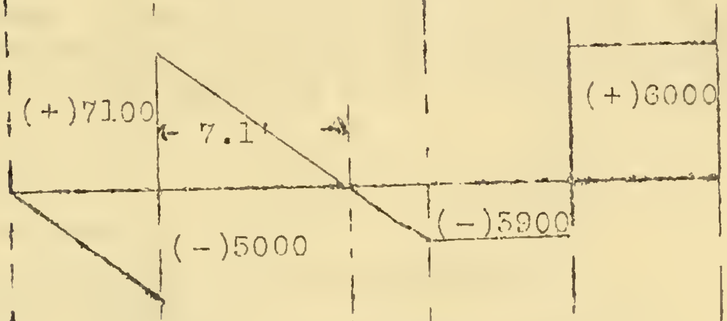


Fig.31e Curve of Shears

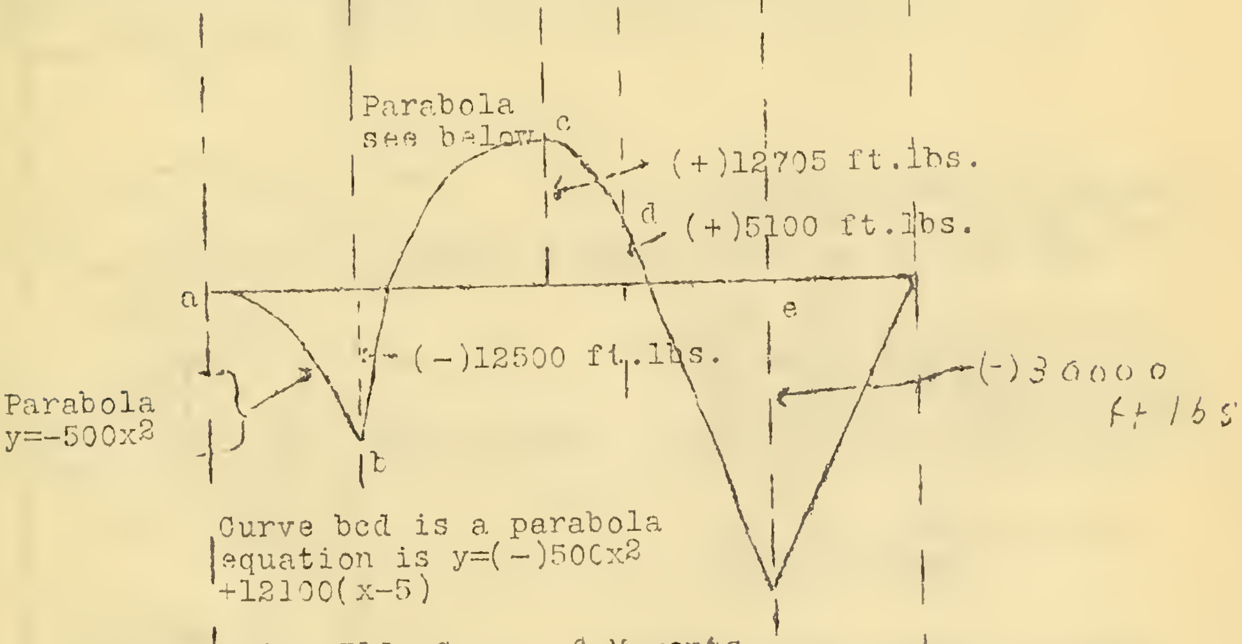


Fig. 31f. Curve of Moments.

Read Articles 32 and 33 before studying this figure.

Note that point of maximum moment occurs where curve of shears crosses the axis. This may be verified by placing the first derivative of the moment with respect to X equal to zero.

32 SHEAR and MOMENT. DISTRIBUTED LOAD. In determining reactions it has been stated that a distributed load may be replaced by its resultant and the latter used as a concentrated load. This method is INCORRECT for shear and moment and should never be used for such cases unless the distributed load lies wholly on ONE SIDE of the section under consideration. The reason for this may readily be seen. Both shear and moment are functions of the forces on ONE SIDE only of a section, and all such forces must be included in the determination of either of these quantities. It is evident that if the structure be loaded with a distributed load its resultant may act on either side of a given section, say on the right, while a considerable portion of the actual load may be on the left. If then the shear or moment be computed for the forces on the left of the section with the distributed load replaced by its resultant the serious error of neglecting a considerable portion of the loads will be made. For reactions, on the other hand, it is the influence of the load as a whole which is to be considered hence the resultant may properly be used. To illustrate the difference between the correct curves of shear and moment for the case of a beam carrying a uniformly distributed load, and the same curves if drawn in accordance with the erroneous assumption that the load may be replaced in magnitude and position by a concentrated load, see Figs. 32a-32b-32c-32d.

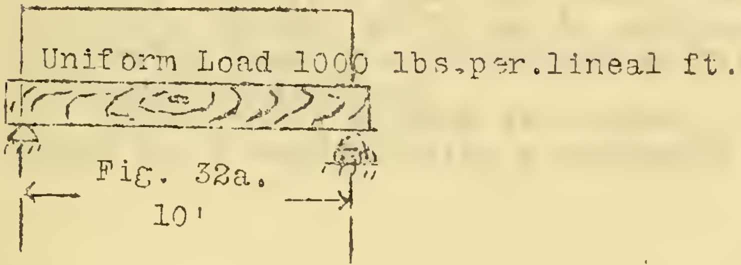


Fig. 32a.

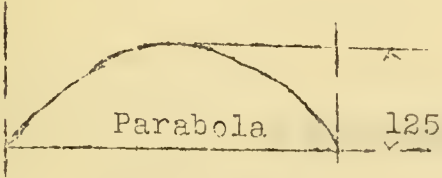


Fig. 32b.

Fig.32b shows the correct curve of moments for beam loaded as in Fig. 32a.

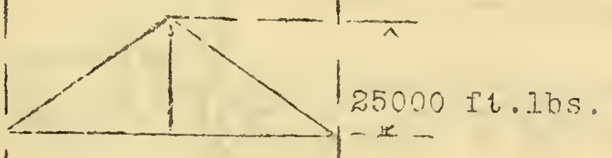


Fig. 32c.

Fig. 32c shows the curve of moments drawn by assuming that the load is concentrated at centre of beam. This curve is incorrect.

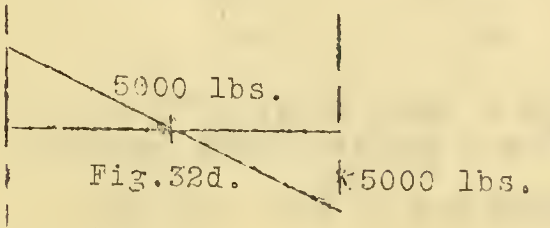


Fig.32d.

Fig. 32d shows correct curve of shears.

Fig. 32e shows curve of shears assuming that load is concentrated at centre. This curve is incorrect.

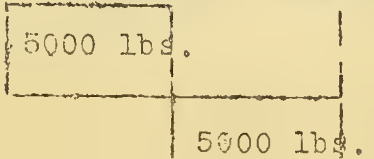
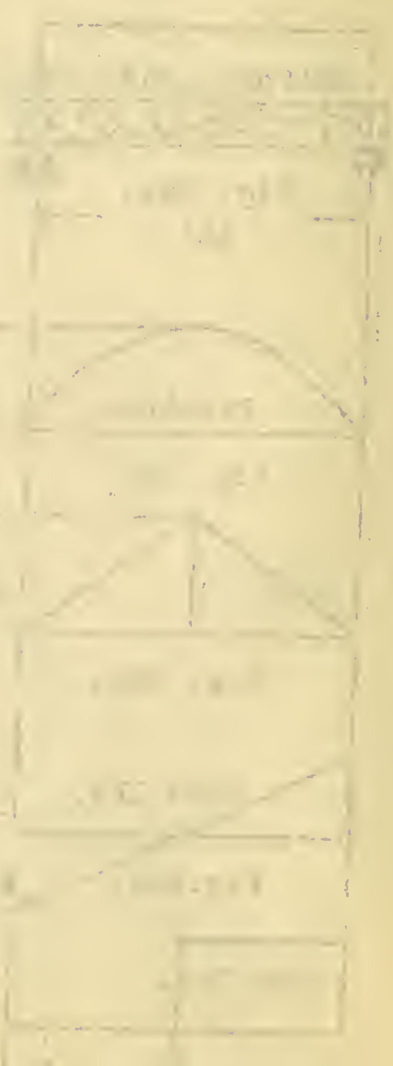


Fig. 32e.

[illegible]



[illegible]

The determination of shear for a distributed load involves nothing but the summation of the loads on either side of the section and is very simple. The determination of moments is more complicated and the following principles of Mechanics may be used to advantage.

1°. The resultant of a series of parallel forces is parallel to the forces, equals their sum, and passes through their centre of gravity.

2° The moment of the resultant of a series of parallel forces about an axis perpendicular to the plane of action of the forces equals the algebraic sum of the moments of the individual forces about the same axis.

The application of the rules just given to the case of a beam carrying a uniformly varying distributed load is shown in the following problems. For cases where the load does not vary uniformly the same general method of dividing the load into several parts may be adopted. The case of a uniformly varying load is quite common in building work and should be thoroughly understood.

Prob. 32. To draw the curves of bending moments and shear for a beam carrying a uniformly distributed load.

A uniformly varying load

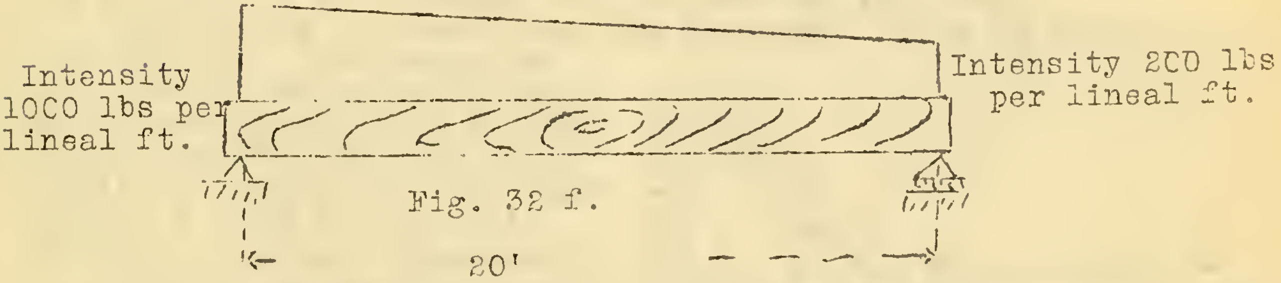

Intensity 1000 lbs per lineal ft.
Intensity 200 lbs per lineal ft.
20'

Fig. 32 f.

Load on above beam is similar to that which would occur on a vertical beam carrying a gate exposed to water pressure.

Let the load be represented in intensity by the trapezoid,

a b c d, shown in Fig. 32g, the area of which equals the total load on the beam. If the trapezoid be divided into two parts by a line c e parallel to the axis a d the effect of each portion may be treated separately and the problem simplified.

Magnitude of force represented by triangle
$$bce = \frac{800}{2} \times 20 = \qquad 8000 \text{ lbs.}$$
Magnitude of force represented by rectangle adce = 200 x 20 = 4000 "

Total load = 12000 lbs.
= area of trapezoid abcd.

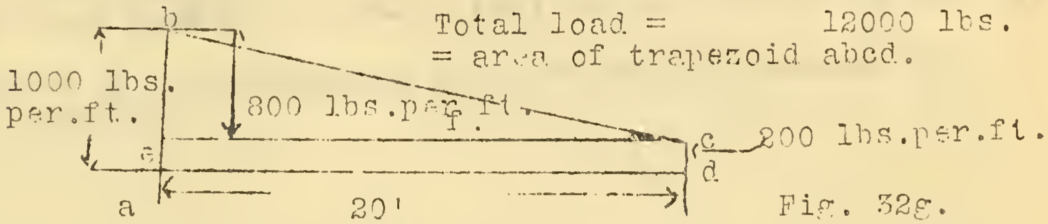


Fig. 32g.

Computation of Reactions. Left reaction $V_L{}^1$, due to loads represented by aced $= \frac{200 \times 20}{2} = 2000$ lbs. To compute reactions due to loads represented by triangle bce it is advisable to determine the position of the resultant of these loads. This passes through the centre of gravity, f, of the triangle (See 1°) and hence is $\frac{20}{3}$ ft from the line ab and $\frac{40}{3}$ ft. from cd.

The left reaction, V_L", due to loads, bce, may be determined by applying $\Sigma M=0$ about right hand end of beam. The following expression results-$8000 \times \frac{40}{3} - V_L \times 20 = 0$, hence $V_L" = 5333$ lbs. The total left reaction, V_L, equals $V_L" + V_L'$ = 7333 lbs.

To obtain the right reaction apply $\Sigma V = 0$. This gives $7333 - 12000 + V_R = 0$ hence $V_R = 4667$ lbs which may be checked by applying $\Sigma M = 0$ about the left end of the beam.

The curve of shears may now be drawn. This is shown in Fig. 32h. Its equation referred to rectangular axes passing through point d with x positive to the left and y positive upwards is $Y = -4667 + 200x + 20x^2$ in which the term 200 x equals the area of a ractangle of height cd and length x (See Fig. 32g) and the term $20x^2$ equals the area of that portion of the triangle bce comprehended between its vertex, c, and a vertical

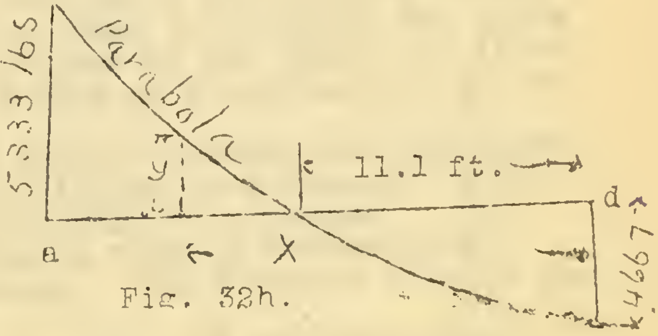
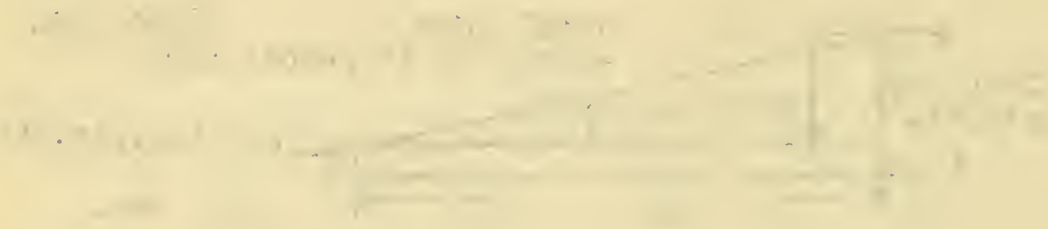


Fig. 32h.

Curve of shears for beam shown in Fig. 32g.



line drawn at a distance x from the vertex. This curve cuts the axis at a point 11.1 ft. from right end as may be seen by placing $y = 0$ and solving for x.

The curve of moments may be obtained in a similar manner and is given in Fig. 32^1. Its equation referred to same origin is $y = 4667x - \frac{200x^2}{3} - \frac{20x^3}{3}$. This equation may be written directly from the shear equation by multiplying each term in the latter, which represent forces, by the distance of the particular force from the section. Thus 4667 is the right reaction and hence should be multiplied by X; 200x is the area of a rectangle of length = x and height = 200 and its moment about its left end equals the area multiplied by 1/2 its base or $\frac{X}{2}$; the area of the triangular portion represented by $20X^2$ is multiplied by $\frac{X}{3}$, the distance between its left end and its centre of gravity.

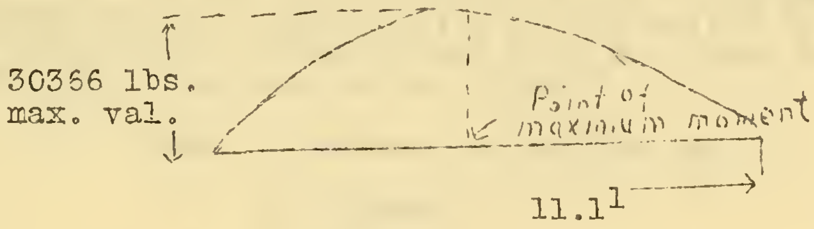


Fig. 32 i

Curve of moments for beam shown in Fig. 32f.

33 LOCATION OF POINT OF MAXIMUM MOMENT. The maximum moment always occurs at the point where the curve of shears cuts the horizontal axis. The truth of this proposition depends upon the fact that the first derivative of the moment equals the shear (see any standard work on Mechanics). In order that the moment may be a maximum or a minimum its first derivative must equal zero, hence the shear must equal zero. As this occurs wherever the curve of shears crosses the horizontal axis its maximum moment must occur at such a point. In the case in question the curve of shears also cuts the axis at each end of the beam, but it is evident that the moments at such points is a minimum and not a maximum.

The reader will observe that if the equation for the curve of moments in article 32 be differentiated with respect to X the equation for the curve of shears will be obtained. In the light of what has just been stated this is correct and such a result should always be found. The converse of this is also true, viz;

That the moment curve is the integral of the shear curve with respect to X and hence the ordinate to the curve of moments at any point equals the area of the shear curve between

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

the end of the beam and the section. An inspection of the numerous shear and moment diagrams on the following pages will show that this relation occurs in every case. The student in testing this by integration must not forget the constant of integration.

34 THEOREM for COMPUTING MOMENTS. In computing moments at a number of consecutive points as is often necessary in dealing with concentrated loads, the following theorem may be used to great advantage;-

The moment at any section b of a structure loaded with parallel forces, either concentrated or distributed, is equal to the moment at any other section a, at a distance X from b, plus (algebraically) the shear at a multiplied by X, plus (algebraically) the moment about b of the loads between a and b. This may be expressed as follows;

Let S_a = shear at section a.
" M_a = moment " " "
" M_b = " " " b.
X = distance between section a and section b measured at right angles to the line of action of the forces.
" M_x = moment about b of forces between a and b.

Then $M_b = M_a \pm S_a \pm S_aX \pm M_x$.

This may be proven in the following manner.

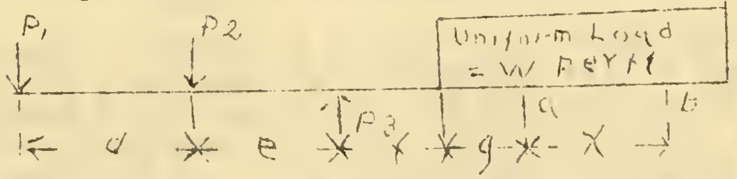


Fig 34a

$$M_a = -P_1\ (d+e+f+g) - P_2\ (e+f+g) + P_3\ (f+g) - \frac{wg^2}{2}$$

$$M_b = -P_1\ (d+e+f+g+x) - P_2\ (e+f+g+x) + P_3\ (f+g+x) - w\ \frac{(g+x)^2}{2}$$

$\therefore$ by subtraction $M_b - M_a = -P_1X - P_2X + P_3X - \frac{w}{2}(2gx + x^2)$

But $P_3 - P_1 - P_2 - wg = S_a$ and $\frac{wx^2}{2} = MX$.

$\therefore$ $M_b = M_a + S_aX - M x$.

This solution is perfectly general since no restrictions were imposed upon character or position of the loads.

35 BEAMS FIXED AT ENDS. The beams hitherto dealt with have been supported at two points and have been statically determined. Sometimes, however, beams are used which are held at the ends by being built into the masonry or otherwise and are statically undetermined. Complete treatment of such beams may be found in standard books on Mechanics and will not be repeated here, although attention should be called to the fact that such beams are much stronger than beams of the same size which are merely supported at the ends.

A beam fixed at one end is also indeterminate with respect to the reactions but the moment and shear at any section of the projecting end can be computed without difficulty.

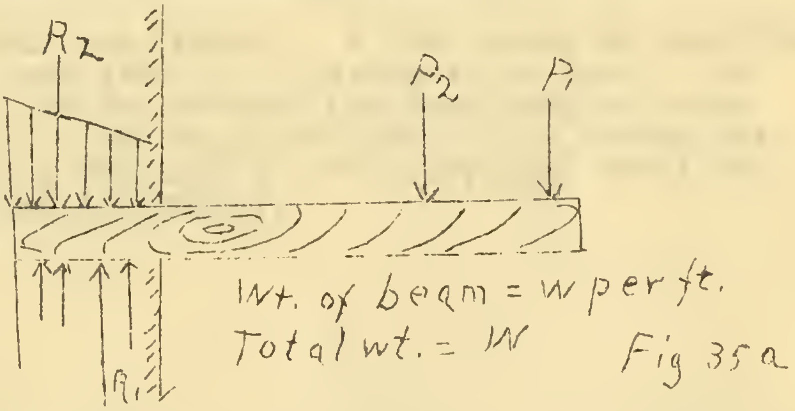


Fig 35a

Such a beam is shown in Fig. 35a in which an assumed distribution of the reactions is indicated, viz; a uniformly varying downward reaction on the top the resultant of which is R_2, and another uniformly varying reaction on the bottom with a resultant equal to R_1. It is evident that $R_1 = R_2 + P_1 + P_2 + W$, and that the moment of R_2 about the point of application of R_1 must equal the moment about the same point of P_1, P_2, and W. The actual distribution of the reaction depends upon the relative elasticity of beam and masonry and will not be discussed. The maximum bending moment and shear occur at, or very near, the edge of the masonry and can be computed with no greater error than for ordinary beams resting on masonry abutements, hence a beam of this sort can be designed with comparative certainty, provided reasonable provision is made at the ends for carrying the reactions.

EFFECT OF FLOOR BEAMS. Reactions, moments, and shears upon a structure as a whole are uninfluenced by the internal construction. For example the reactions at the ends of a structure due to a given loading are the same whether it is a simple beam or is made up of trusses, floor beams, and stringers, This immunity however does not extend to the individual members of the structure which are influenced to a marked degree by the construction adopted. In the case of an ordinary bridge composed of trusses or girders, floor beams, and stringers, the shear and moment on the trusses vary considerably from that which would exist if there were no floor beams, and this applies also to the reaction if floor beams be not used at the ends.

The action of floor beams is to load the main girders or trusses with loads at definite points. This is clearly shown by the figures accompanying Article I. The load reaches the stringers through the floor, is carried by them to the floor beams, and thence goes to the main girders. In consequence the girders carry only concentrated loads except for their own weight, and the curves of shear and moment for the applied loads are made up of straight lines.

TYPICAL CURVES of SHEAR and MOMENT. A few curves of shear and moment have already been drawn to illustrate the text. In the figures which follow the attempt has been made to cover a wide range of cases. The beginner should draw curves for similar cases, changing the data to avoid copying, until he understands the subject thoroughly.

Typical Curves of Shears and Moments.
Beam fixed at one end.

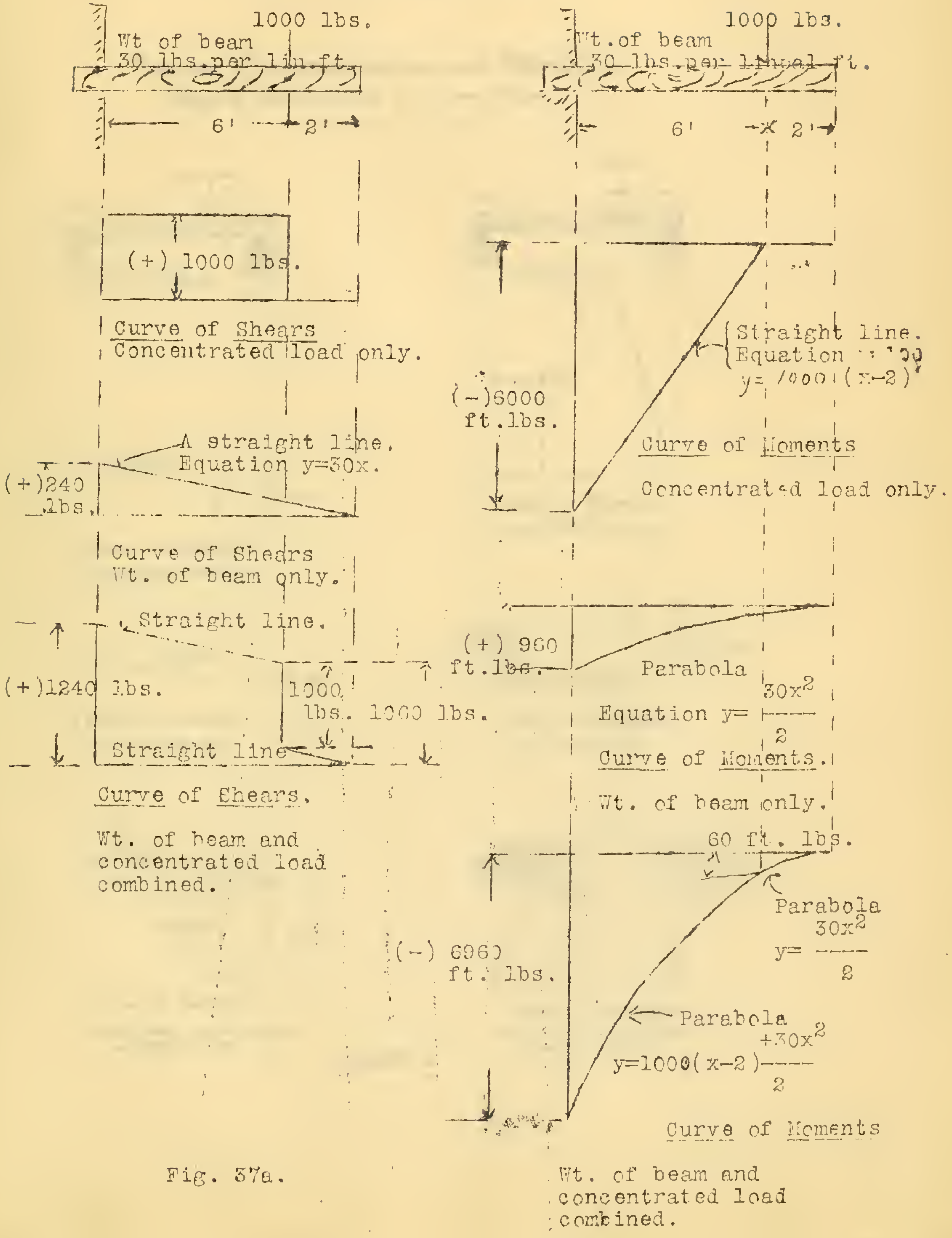
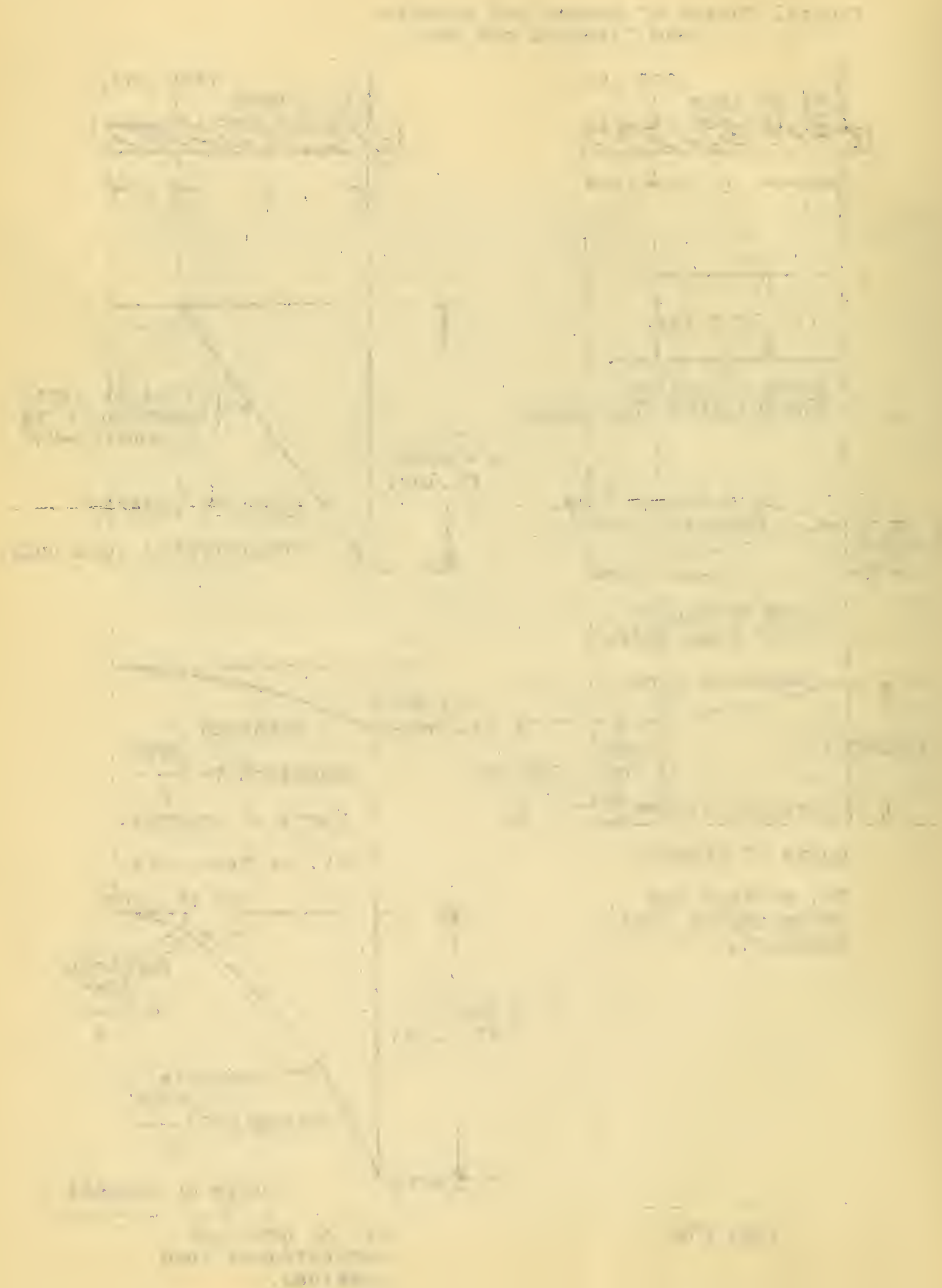


Fig. 37a.



TYPICAL CURVES OF SHEARS AND MOMENTS.

BEAMS SUPPORTED AT BOTH ENDS.

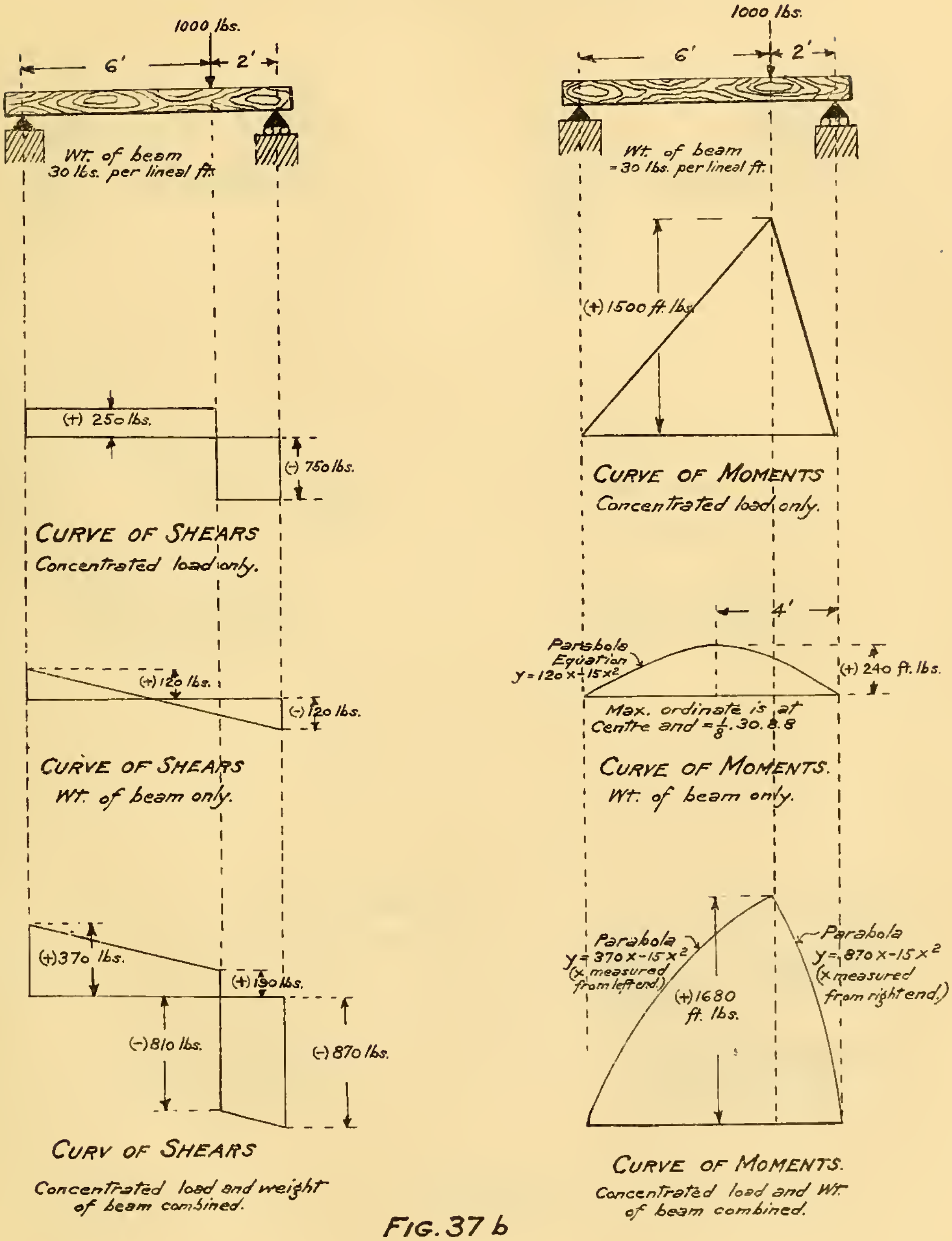

1000 lbs.
6'
2'
Wt. of beam 30 lbs. per lineal ft.
(+) 250 lbs.
(-) 750 lbs.
CURVE OF SHEARS
Concentrated load only.
(+) 120 lbs.
(-) 120 lbs.
CURVE OF SHEARS
Wt. of beam only.
(+) 370 lbs.
(+) 190 lbs.
(-) 810 lbs.
(-) 870 lbs.
CURV OF SHEARS
Concentrated load and weight of beam combined.
1000 lbs.
6'
2'
Wt. of beam = 30 lbs. per lineal ft.
(+) 1500 ft. lbs.
CURVE OF MOMENTS
Concentrated load only.
4'
Parabola Equation $y = 120x - 15x^2$
(+) 240 ft. lbs.
Max. ordinate is at Centre and $= \frac{1}{8} . 30 . 8 . 8$
CURVE OF MOMENTS.
Wt. of beam only.
Parabola $y = 370x - 15x^2$ (x measured from left end.)
Parabola $y = 870x - 15x^2$ (x measured from right end.)
(+) 1680 ft. lbs.
CURVE OF MOMENTS.
Concentrated load and Wt. of beam combined.

FIG. 37 b

TYPICAL CURVES OF SHEARS AND MOMENTS.

BEAM FIXED AT ONE END.

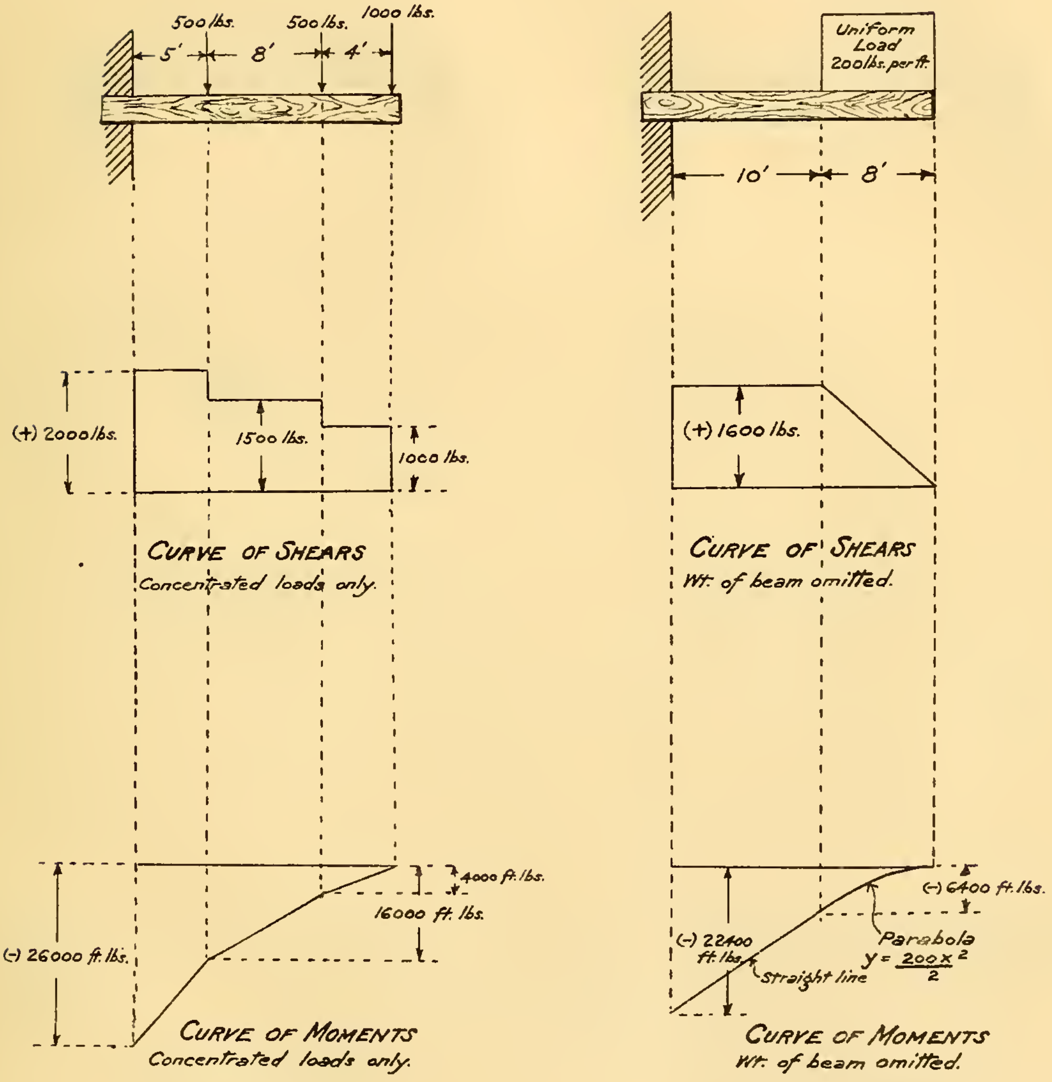
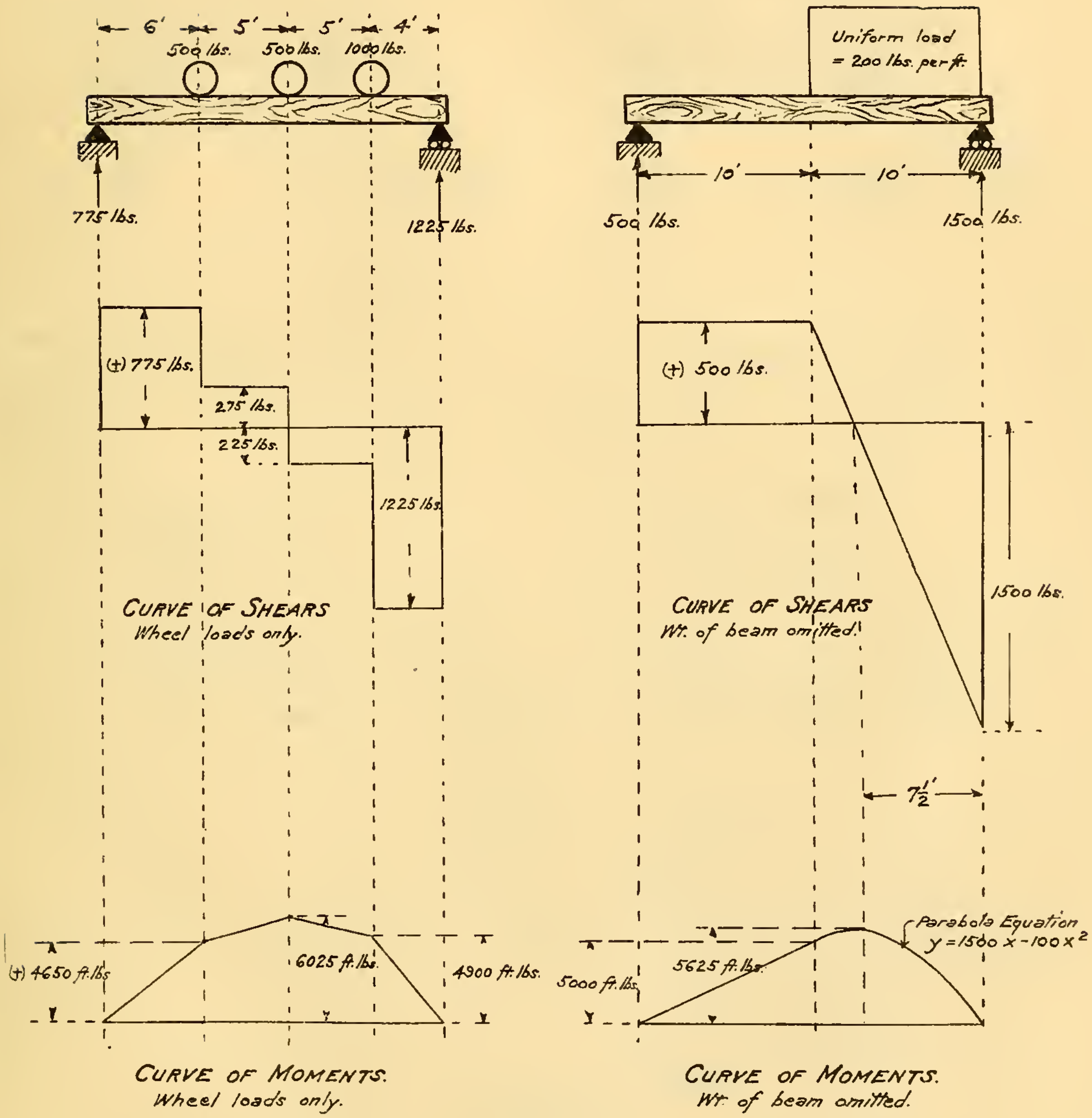


FIG. 37 C

TYPICAL CURVES OF SHEARS AND MOMENTS.
BEAM SUPPORTED AT BOTH ENDS.



CURVE OF MOMENTS.
Wheel loads only.

CURVE OF MOMENTS.
Wt. of beam omitted.

FIG. 37d

TYPICAL CURVES OF SHEARS AND MOMENTS.

BEAM SUPPORTED AT TWO POINTS.

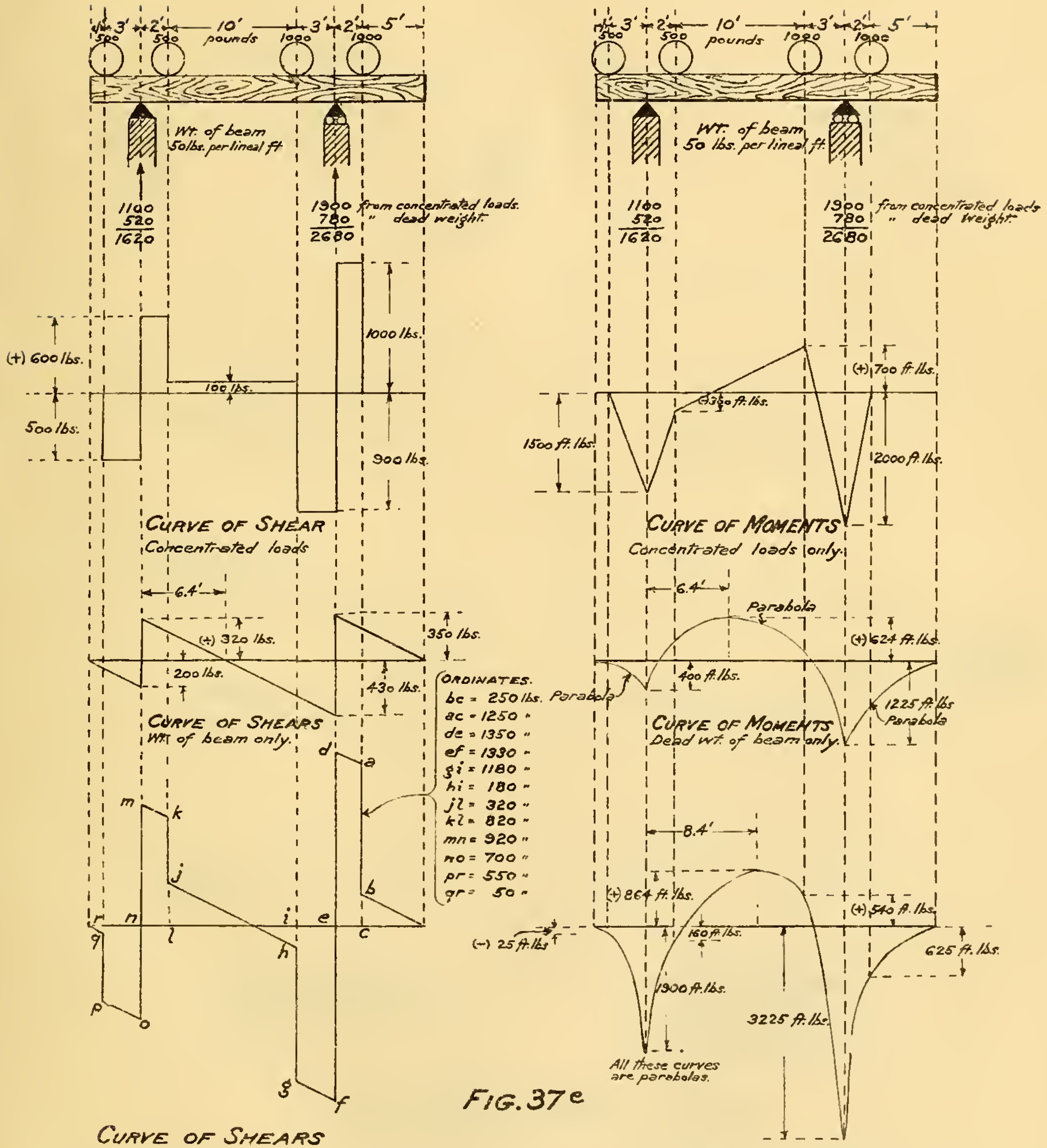


FIG. 37e

CURVE OF SHEARS

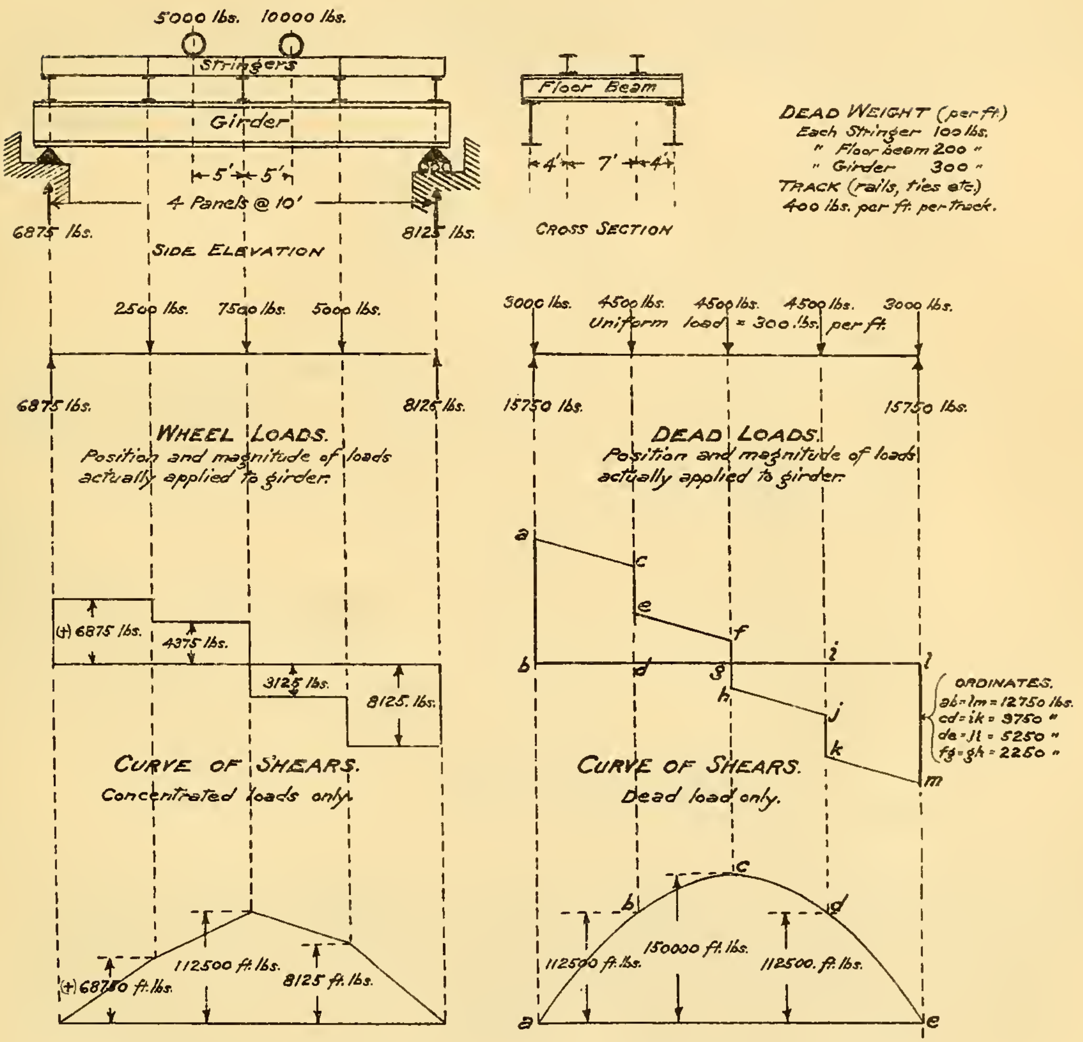
Concentrated loads and wt. of beam combined.

CURVE OF MOMENTS.

Concentrated loads and wt. of beam combined.

TYPICAL CURVES OF SHEARS AND MOMENTS.

END SUPPORTED GIRDER WITH FLOOR BEAMS.



CURVE OF MOMENTS.
Concentrated loads only.
Moment at Panel points is same as if the loads were applied directly to the Girder.

CURVE OF MOMENTS.
Dead load only.
Locus of points a, b, c, d and e is a parabola - but moment curve is not a parabola.

NOTE:— Floor beams are ordinarily riveted to sides of girders. Above construction is adopted here for sake of clearness.

FIG. 37f

38 INFLUENCE LINES and TABLES DEFINED. In the determination of maximum shears, moments, reactions, and other functions due to moving loads it is frequently useful to study the effect of a load of unity as it moves along the structure. This may be done graphically by plotting a line called an influence line to show the effect of the load, or analytically by preparing an influence table in which are set down the values of the function under consideration when the load is at various governing points such as the panel points of a truss bridge. The following simple examples shows clearly the character of line and table.

Influence line for
Simple beam.
Shear at a

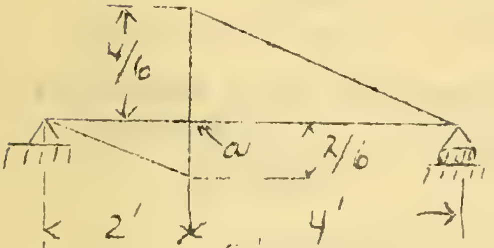


Fig 38a

Distance of load from right reaction	Shear at a
1 ft.	+ 1/6
2 ft.	+1/6
3 ft.	+3/6
3.9 ft.	+ 39/60
4.1 ft.	- 19/60
5 ft.	- 1/6

Influence Table
for beam shown in Fig.38a.

The influence line is the locus of the values in the second column of the influence table and is merely the graphical representation of the equation for the shear at a due to a load of unity passing along the beam. If X be the distance of the load from the left reaction and Y the ordinate the equations of the influence line will be as follows;-

$$Y = -\frac{X}{6} \qquad X \text{ varying between } 0 \text{ and } 2'$$

and

$$Y = \frac{6-X}{6} \qquad X \quad '' \qquad '' \quad 2' \text{ and } 6'$$

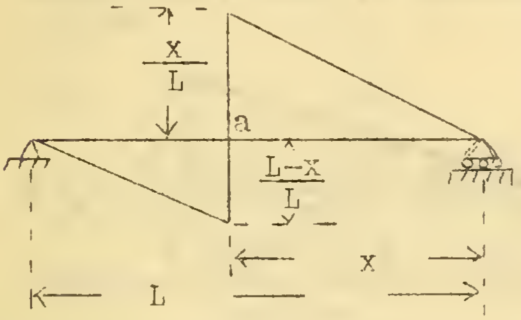
The difference between an influence line and the curves described in the preceding articles should be carefully observed. A curve of shears or moments is a curve the ordinate to which at any point shows the shear or moments at that point caused by the set of loads, fixed in magnitude and position, for which the curve is drawn. The ordinate to the influence line shows instead the shear or moments at the section for which the influence line is drawn, due to a load of unity acting

at the point where the ordinate is measured. The example in Article 39 serves to illustrate influence lines for the more common cases of simple beams and girders.

The actual employment of influence lines and tables in practice seldom occurs except for complicated structures where they are frequently almost indispensable. In this book the influence line will, however, be used with freedom, partly for purposes of illustration and demonstration and partly that the student may better familiarize himself with the behavior of various structures under moving loads.

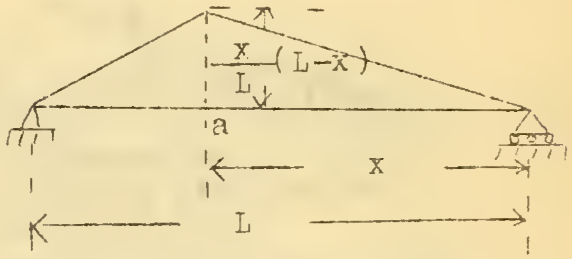
39. EXAMPLES OF INFLUENCE LINES.

Positive values are shown above axis in all cases.



Influence Line for Shear at Section a.

Fig. 39a.



Influence Line for Moment at Section a.

Fig. 39b.

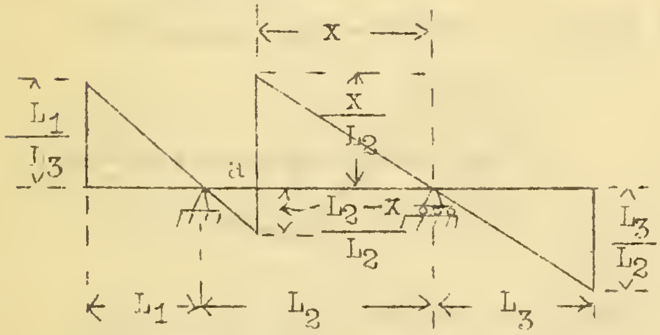


Fig. 39c.

Influence Line for Shear Sec. a.

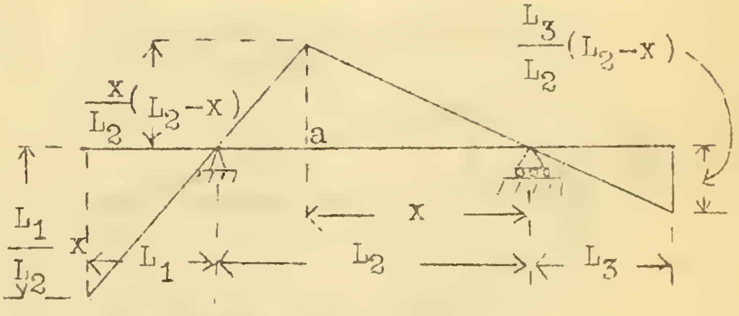


Fig. 39d.

Influence Line for Moment Section a.

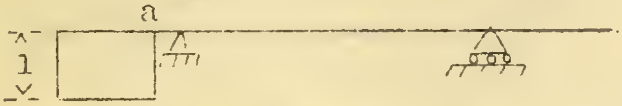


Fig. 39e

Influence Line for Shear at Section a.

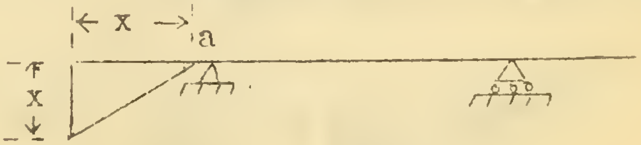


Fig. 39f.

Influence Line for Moment at Section a.

Figs. 39a to 39f inclusive are for simple beams, or girders, ithout floor beams. Influence lines for girders with floor ams follow.

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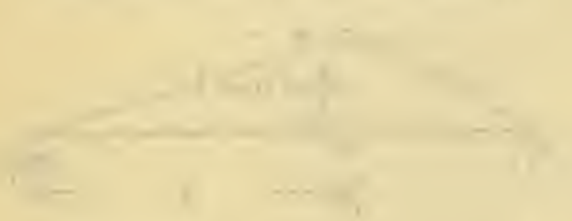

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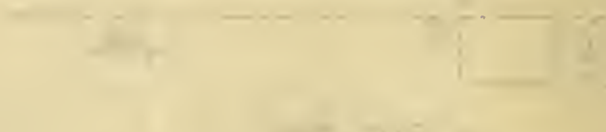

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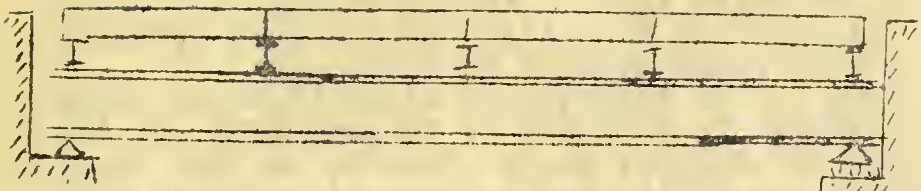
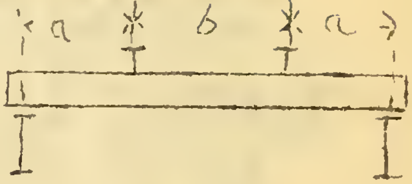
See note upon type of construction, Fig. 37h.

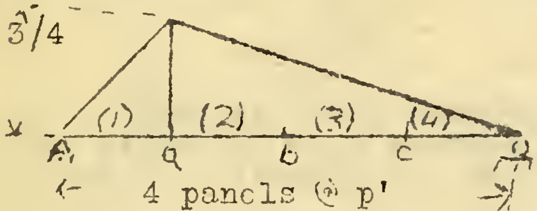


Side Elevation of Bridge
4 panels @ p' = 4p' = span

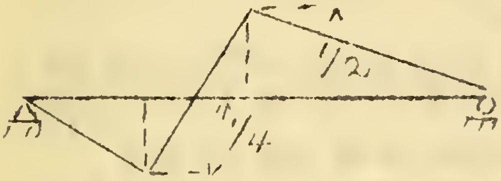


Cross Section
of a bridge

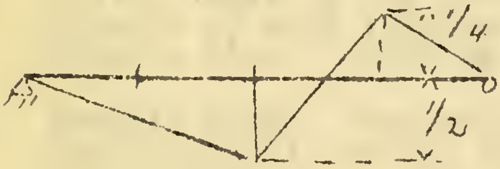
Influence lines which follow are for one girder of above bridge and show the effect of two loads of unity moving along the structure, one upon each stringer, Since the bridge is symmetrical this is equivalent to one load of unity for each girder.



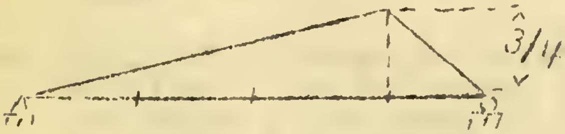
Influence Line-Shear in Panel (1)



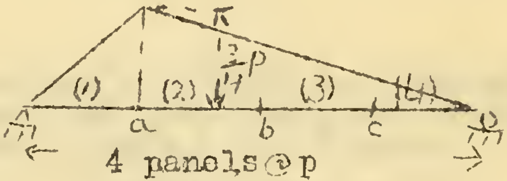
Influence Line-Shear in Panel (2)



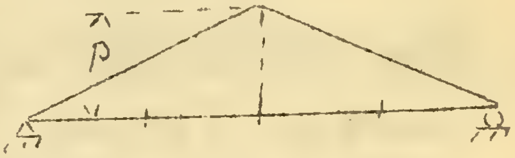
Influence Line-Shear in Panel (3)



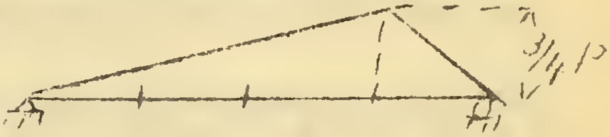
Influence Line-Shear in Panel (4)



Influence Line-Moment at Panel point A.

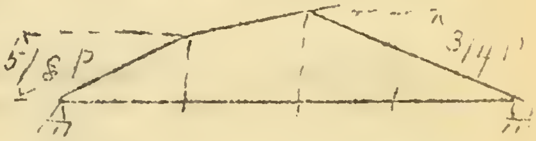


Influence Line-Moment at panel point b.



Influence Line-Moment at panel point c.

Fig. 39g.



Influence Line-Moment at centre of panel (2)

40 PROPERTIES OF THE INFLUENCE LINE. The following theorems may often be used to advantage.

1° The value of a given function due to a single load in a fixed position equals the product of the magnitude of the load

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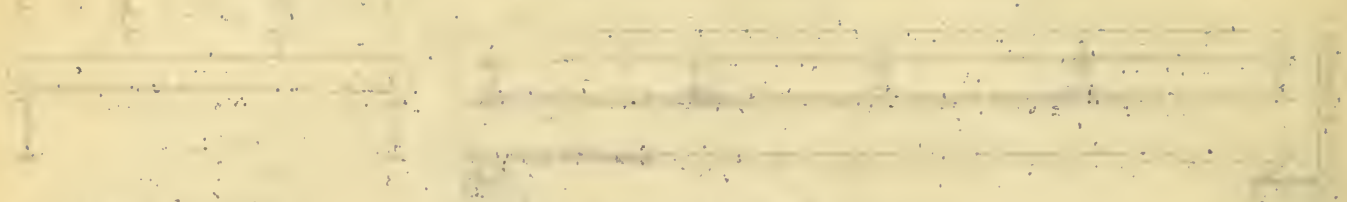

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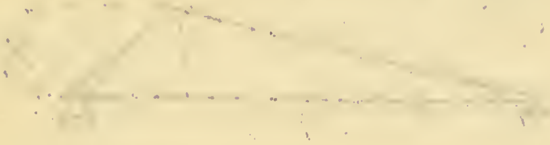

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[illegible]

and the ordinate to the influence line measured at the point where the load is placed. This needs no proof but follows directly from the definition of the influence line.

2° The value of a given function due to a uniformly distributed load equals the product of the INTENSITY of the load and the AREA bounded by the axix of the beam, the influence line, and the ordinate drawn through the limits of the load. This area may be partially positive and partially negative. The algebraic sum of the two should be used.

This theorem may be proven as follows;-

Let bcd represent an influence line for a portion of a given structure of length L. Let w equal the intensity of a uniformly distributed load.

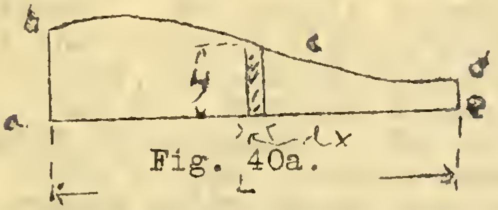

Fig. 40a.

Then the total load on a section of length dx = wdx and the effect of this portion of the load upon the given function = wydx. Integrating between the limits 0 and L gives

$$w\int_0^L ydx$$

as the effect of a load covering the entire distance L. But ydx is the area of the infifinitesimal strip subtended by dx and $\int_0^L ydx$ is the area abcde, hence $W\int_0^L ydx = W \times$ area abcde.

3° The value of a given function due to a set of concentrated loads in any position equals the algebraic sum of the product of each load and its corresponding ordinate to the influence line. This is a corollary of 1°.

41 NEUTRAL POINT. The influence lines for shear shown in Fig. 39g cross the axis of the beam in each case except that for shear in the end panels. This point of intersection is called the neutral point since a single load placed at this point produces no shear in the panel where the intersection occurs.

The neutral point for the end panels is at the ends of the beams.

42. POSITION OF LOADS FOR MAXIMUM SHEAR and MOMENT at a DEFINITE SECTION. The following important laws may be derived from the influence lines given in Article 39.

1°. In a single beam supported at the ends a single concentrated load causes maximum shear at a section when placed an infinitesimal distance on one or the other side of the section, and maximum moment when placed at the section. A uniformly distributed live load produces maximum shear at a section when applied over the entire distance between the section and one or the other end of the beam, and maximum moment when applied over the entire length of the beam.

2°. In a girder or truss loaded by means of floor beams, a single concentrated load produces a maximum shear or a maxi-

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[illegible]

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mum moment in a panel when placed at the floor beam at one or the other end of the panel, and the maximum moment at a panel point when placed at that point. A uniformly distributed live load produces a maximum shear in a panel when applied over the entire distance between the neutral point of that panel and one or the other end of the structure, and a maximum moment at any point when applied over the entire length of the structure.

43 MAXIMUM MOMENTS and SHEARS--STRUCTURES SUPPORTED AT ENDS.

In the preceding article moments and shears at particular sections have alone been considered, and no attention has been given to the maximum values of these functions. These maximum values must, however, be computed before the structure can be designed. For single concentrated loads and for uniform live load the value of these quantities can be easily determined as follows, for beams supported at ends.

Case 1. Maximum shear, single concentrated load, beam without floor beams, The influence line shows that the maximum value of the ordinate occurs either when $X = L$, or $L-X = L$; and equals unity, hence the maximum shear in this case under a concentrated load of magnitude P, equals P, and occurs with the load at either end of the beam.

Case 2. Maximum moment on beam under same conditions as Case 1. Here the ordinate to the influence line is a maximum at the load and equals $\frac{X}{L}(L-X)$. This can be easily shown to be a maximum when $X = L-X$, hence the maximum moment due to a load P occurs when the load is at the centre of the beam. Its value is $\frac{PL}{4}$.

Case 3. Maximum shear on same beam as Case 1 due to a uniform live load of intensity w. It is evident that the area between the influence line and the axis will be a maximum if section a is at either end, hence the maximum shear equals $\frac{WL}{2}$.

Case 4. Same as Case 3 but maximum moment instead of shear. The maximum moment occurs for load over entire beam and at the section where the ordinate is a maximum which has already been shown in Case 2 to be at the centre. The moment at the centre equals $1/8\ wL^2$.

Case 5. Maximum shear. Single concentrated load. Girder with floor beams. Panel lengths equal. The maximum evidently occurs in the end panel; its value depends upon the number of panels. If n equals number of panels and P the load the maximum shear $= \frac{P\ (n-1)}{n}$.

Case 6. Same as Case 5 but uniform load w per ft instead of concentrated load. Maximum shear occurs in end panels and equals $\frac{wp}{2}(n-1)$ where $p =$ panel length.

Case 7. Same as Case 5 but maximum moment instead of maximum shear. Place load at panel point nearest centre. Maximum moment occurs at this panel point and equals $\left(\frac{P}{2}\right)\left(\frac{pn}{2}\right)$ if number of panels is even, and $\frac{Pp}{4n}(n^2-1)$ if number of panels is odd.

[illegible]

[illegible]

Case 2. [illegible]

Case 3. [illegible]

[illegible]

Case 4. [illegible]

Case 5. [illegible]

[illegible]

Case 6. [illegible]

Case 7. [illegible]

Case 8. Same as Case 6, but maximum moment instead of maximum shear. Maximum moment occurs at panel point nearest centre with load over entire span. Its value is $\frac{1}{8}WL^2$ when number of panels is even and $\frac{1}{8}WL^2 \left(1-\frac{1}{n^2}\right)$ when number is odd. (See Fig. 43b).

In deriving these two quantities the following theorem now used:-

"The moment at a panel point of a girder with floor beams equals that at the corresponding point of a simple beam under the same load.

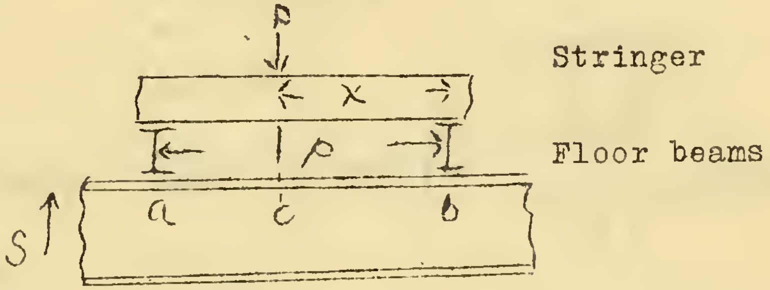


Fig. 43a.

The proof of the theorem is as follows;-

Let Fig. 43a represent a portion of a girder carrying floor beams.

Let M_b = moment at panel point b.

" M_a = " " " " a.

" S = shear in panel to left of given panel.

Then in accordance with rule given in article 34

$M_2 = M_1 + S_p - P\left(\frac{X}{P}\right)P = M_1 \; S_p - Px$ which is the value of the moment at (b) eith the load P applied directly to the girder at the point C.

Of the formulas in this article the student is advised to memorize that for maximum moment at the centre due to a uniform load, viz; $M = \frac{1}{8}WL^2$. This is applicable both to simple beams and girders with floor beams and an even number of equal panels.

Since the moment at a panel point equals that at the corresponding point of a simple beam under the same load the locus of the moments at the panel points is a parabola, with a centre ordinate equal $\frac{1}{8}WL^2$, hence the ordinate at any panel point of a girder with an odd number of panels may be deduced from this value by applying the equation of the parabola. This is illustrated by the following figure;-

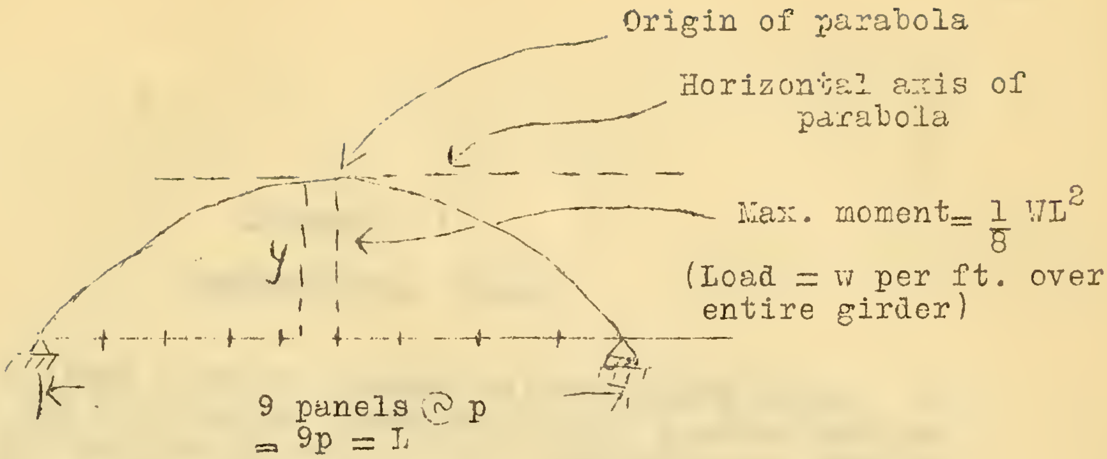

Origin of parabola
Horizontal axis of parabola
Max. moment = $\frac{1}{8}$ WL^2
(Load = w per ft. over entire girder)
y
9 panels @ p
= 9p = L

Fig. 43b.

The ordinate y equals $\frac{1}{8}\ WL^2 \left(1 - \frac{\left(\frac{p}{2}\right)^2}{\left(4\frac{1}{2}\ p\right)^2}\right)$ = maximum moment on girder.

44. APPROXIMATE METHOD for MAXIMUM SHEAR. In practice it is common to determine maximum shear produced by a uniform load on girders with floor beams by the following approximate but safe method.

Compute the maximum positive shear in a panel as if all panel points to right were loaded with full panel loads and panel points at left with no load; for maximum negative shear reverse this process.

This method is illustrated by the following example.

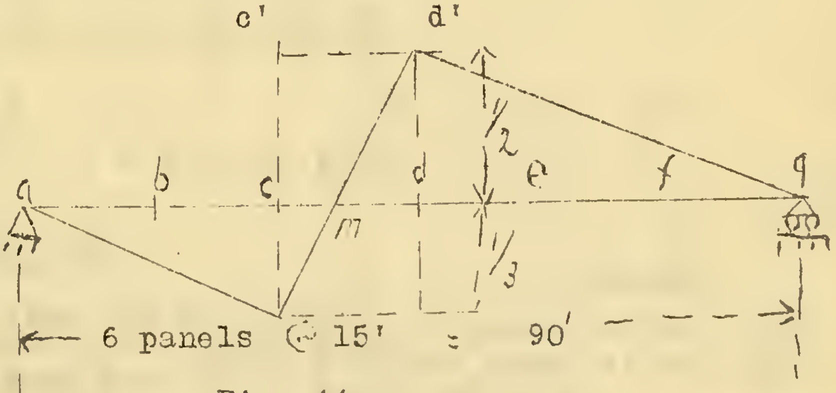


Fig. 44a.

The position of the neutral point may be found by similar triangles $\frac{m\ d}{c\ d} = \frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{3}}$ ∴ $m\ d = \frac{3}{5}\ c\ d = 3/5\ (15) = 9'$

CHAPTER III.

CONCENTRATED LOADS.

45. SHEAR at a FIXED SECTION --GIRDER without FLOOR BEAMS. To determine the position of loads which will produce maximum shear at a given section of a simple end-supported beam or deck-girder a method of trial may be employed. Stated briefly this consists of moving the loads along the beam by intervals corresponding to the distance between wheels and writing expressions for the change in shear thus produced. This process is repeated until the shear is found to decrease.

This method is based upon the fact that the maximum shear at a given section of a simple beam carrying concentrated loads occur with one of the loads at an infinitesimal distance from the section.

The proof of this proposition and the application of the method to a definite case will now be given.

Let Fig. 45a represent a typical set of concentrated loads, in this case a single consolidated locomotive, and Fig. 45b a beam for which it is desired to compute the maximum shear at section a.

Loads in thousands of lbs. per wheel.

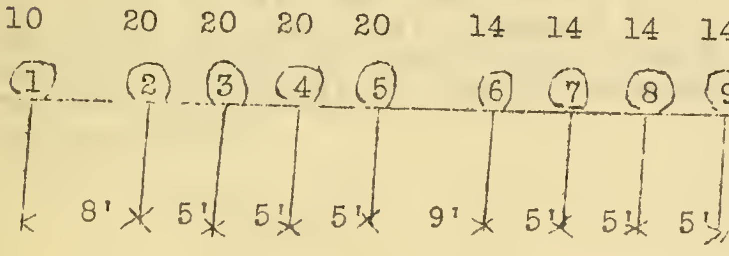


Fig. 45a

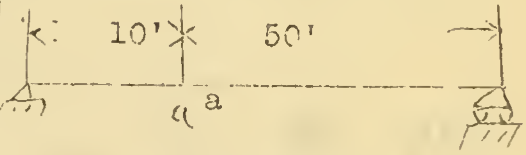


Fig. 45b

The influence line for the section is shown in Fig. 45c and shows clearly that for maximum positive shear at section a most of the heavy loads must be to the right of a.

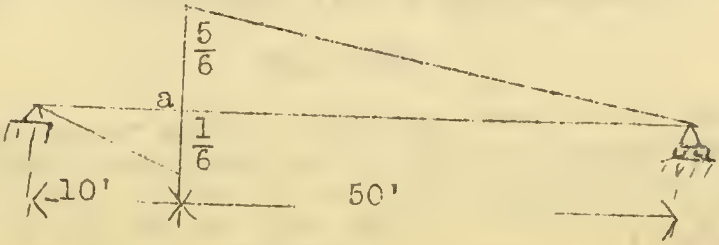
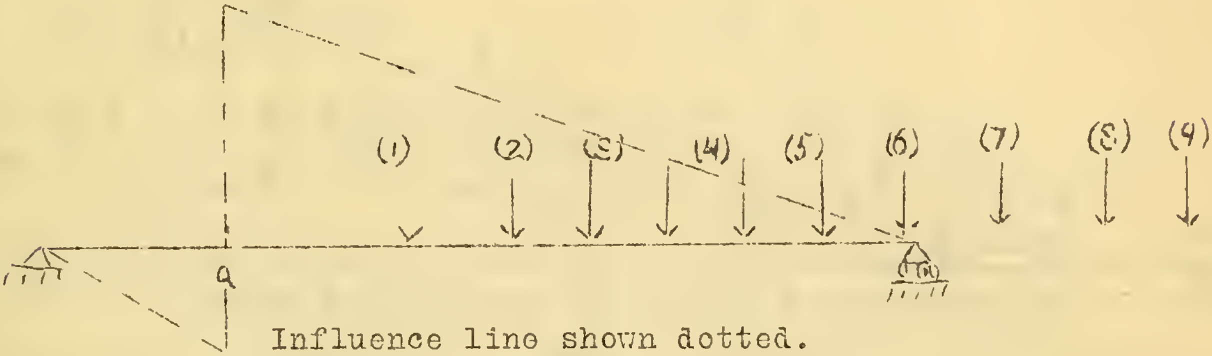


Fig.45c.

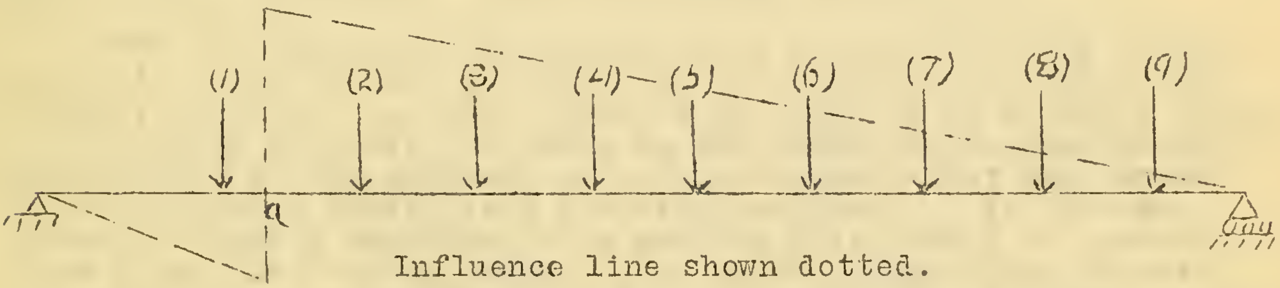
To prove that one of the loads should lie an infinitesimal distance to the right of the section, or practically at the section, proceed as follows. Suppose the loads to be on the beam as shown in Fig. 45d.



Influence line shown dotted.

Fig. 45d.

As the shear due to a set of concentrated loads in any position equals the summation of the product of loads and their ordinates it is evident that starting with loads in above position shear at a will be increased by moving the loads to the left until 1 reaches the section. Ifthe loads are moved still further until load 1 passes to the left of the section there will be a sudden decrease in the shear due to load 1 crossing the section. The new condition is indicated by Fig. 45e from which it is again evident that if the loads be moved still further to the left there will be another increase in shear until load 2 comes to the section, and that the result of load 2 crossing the section will be another sudden



Influence line shown dotted.

Fig. 45e.

decease in shear after which the shear will again increase till another load reaches the section, and so on. It is also clear that the effect of a load coming on the span at the right or going off at the left during the progress of moving up will not effect the truth of above statements.

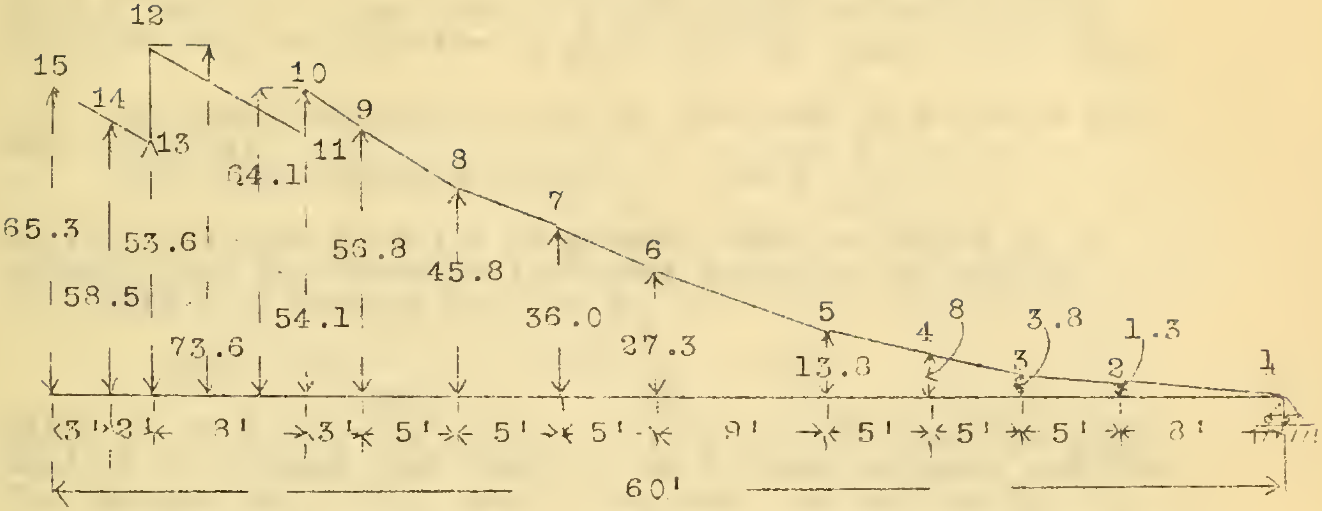


Fig. 45f.

Fig. 45f is a graphical illustration of the changes in shear at a of beams shown in Fig. 45b as the loads move to the left. The ordinates represent this shear with load (1) at the point where the ordinate is shown. In consequence the line 1-2 shows the increase in shear at a due to moving load (1) on the span until load (2) reaches the right end; 2-3 shows the increase due to moving to the left the first two loads until load (3) reaches the right abutment; and so on up to 9-10 which shows the effect of moving the first 9 loads, i.e. all the loads, until the first load reaches the section a. As the first load crosses the section the shear drops suddenly by 10000 lbs. and then increases again as shown by 11-12 until the second load reaches section a. As this load crosses the section the shear is diminished by 20000 lbs. and then increases as shown by 13-14 until the first load passes off the span. This produces no sudden change in shear but changes the slope of the line as shown by 14-15.

From the preceding discussion it is evident that the following method may be used to determine the location of locomotive loads for maximum positive shear at any section of a simple beam:-

Starting with all the loads to the right of the section and with load 1 at the section, write an expression for the change in shear due to moving load 2 to the section. If this expression shows a decrease it is evident that load 1 at the section gives the maximum shear. If on the other hand, the expression shows an increase, it will be necessary to write another expression for the increase due to moving up loads 3 and so on till a decrease is finally obtained.

In practice the operation is simple as is shown by the following example for the beam and loads of this article. It will be noticed that instead of writing an equation for the change in shear the method used is to write an inequality one side of which shows the increase in the left hand reaction

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[illegible]

[illegible]

due to moving up those loads which are on the span to begin with and remain on or which come on during the process of moving, and the other side of which shows the effect of a load crossing the section or going off the span to the left.

The numerical expressions for the case in question will now be given.

With 1 at section move up 2. $146 \times \frac{8}{60} > 10$

As the left hand quantity is greater than the right it is evident that the shear is increased by moving up load 2,

With 2 at section move up 3.

$$(146 - 10)\frac{5}{60} + \partial + (10)\frac{2}{60} < 20$$

Since in this case the left hand side is less than the right hand it is evident that there is no further increase and that the maximum shear will occur with load 2 at section a.

As the left hand side of the above expression may not be entirely clear a few words of explanation may be added. The first term shows the increase in the left hand reaction due to moving up those loads which are on the span to begin with and which remain on the span. The second term ∂ has already been explained, its value in this case being 0. The third term gives the shear caused by load 1 when load 2 is at section a. This shear being negative and disappearing during the movement on account of the load going off the span an increase in shear is obtained which is, therefore, placed on the left hand side of the inequality.

Having in above fashion determined the position of the loads for maximum shear, it remains simply to compute this shear in the ordinary manner by figuring the left hand reaction and subtracting therefrom the loads between it and the section.

MOMENT at a FIXED SECTION. The method of determining the position of loads for maximum moment differs somewhat from that used in determining the position for maximum shear, and is as follows;-

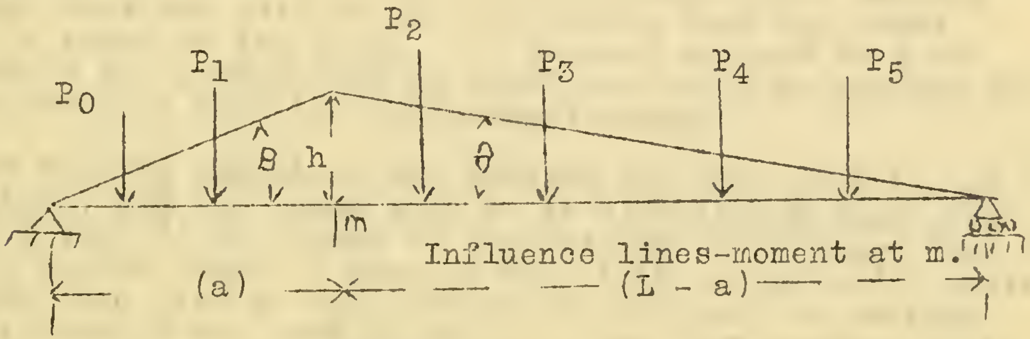


Fig 46a

Let the original position of the loads be as shown in Fig.46a.

Let $\triangle M$ = increase in moment at m due to moving all the loads a short distance d to left.

Then, since the change in moment at m caused by the moment of the load system equals the summation of the product of each load by the change in length of the influence line ordinate corresponding to that load, the following expression for the increase in moment may be written.

$$\begin{aligned}\triangle M &= (P_2+P_3+P_4+P_5)\ d \tan \theta - (P_0+P_1)\ d \tan B \\ &= (P_2+P_3+P_4+P_5)\ d \frac{h}{L-a} - (P_0+P_1)\ d \frac{h}{a}\end{aligned}$$

$$\therefore \quad \frac{\triangle M}{h\,d} = \frac{P_2+P_3+P_4+P_5}{L-a} - \frac{P_0+P_1}{a}$$

This equation shows that the moment at m will be increased by moving the loads to the left if the average load per foot on the right of m be greater than the average load per foot on the left. The converse of this proposition is also true. It should be noted that if the average load per foot on the right equals the average load per foot on the left there will be no change produced in the moment by moving the loads.

The above equations are true so long as the loads do not change; that is, if no load comes on from the right, or goes off to the left, or passes the section. It is readily seen, however, that if the average load per foot on the right exceeds that on the left a movement to the left sufficient to bring another load on from the right or to cause a load to go off to the left serves to increase the moment more rapidly and hence does not vitiate the conclusion that the loads should be moved to the left. It is also evident that the movement to the left should be continued until P_2 reaches the section hence we have the following theorem;-

For maximum moment at any section one load must lie at the section, and the loads must be so distributed that with that load just to the right of the section the average load per foot on the right is greater than that on the left, while with that load just to the left of the section the average load per foot on the left is greater than that on the right.

A special case of the above is when the average load per

foot on one side equals the average load per foot on the other side. In this case a load does not have to lie at the section, but if it does lie at the section the moment will be equal to the maximum, hence the theorem applies for this case also.

The application of this theorem is simple, but it sometimes happens that several loads of the same system will be found to satisfy the above criterion. This is explained by the fact that a different set of loads may be on the span for each position, and consequently there may be several maxima. In such cases it is usually necessary either to compute the value of each maximum, or else to compute the change in moment due to moving the loads from one maximum position to another.

A numerical example of the determination of the position for maximum moment will now be given.

Let the loads and span be as in Article 45 and let the problem be to find the position giving maximum moment at a.

First try load 2	Av. load per ft. on right		Av. load per ft. on left
Load 2 to right of section	$\frac{136}{50}$	$>$	$\frac{10}{10}$
Load 2 to left of section	$\frac{116}{50}$	$<$	$\frac{30}{10}$

Loads 2 gives a maximum

Try load 3.

	Av. load per ft. on right		Av. load per ft. on left
Load 3 to right of section	$\frac{116}{50}$	$>$	$\frac{20}{10}$
Load 3 to left of section	$\frac{96}{50}$	$<$	$\frac{40}{10}$

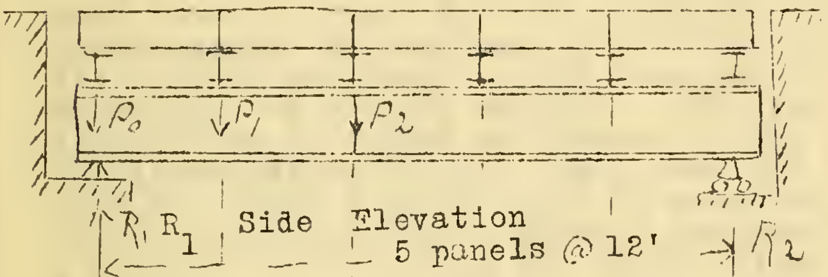
Load 3 also gives a maximum.

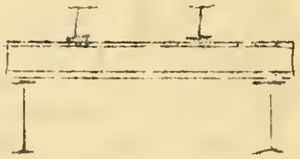
It is seen by inspection that in this case it is unnecessary to try load 4 and that loads 2 and 3 are the only ones giving maximum moments. To determine which of these is the larger it is advisable to compute both independently and then check the results by computing the change in moment produced by moving from load 2 at a to load 3.

That the maximum moment at a given section due to a set of concentrated loads always occurs with a load at the section is apparent from the fact that the maximum moment occurs where the curve shear crosses the axis, and that this can never happen except at one of the loads.

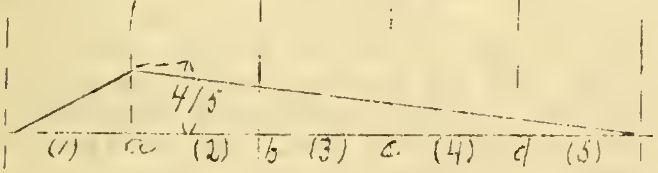
47. SHEAR--GIRDER WITH FLOOR BEAMS. For such girders the maximum shear in every panel must be computed. The method of determining the position of the loads differs in detail from that given in Article 45 although the same general method may be used.

To illustrade this case the bridge shown in Fig.47a is chosen. Here again for greater clearness the stringers and floor beams are shown above the girders though as already explained such construction is uncommon. End floor beams are also used but this makes no difference in the method or its application.





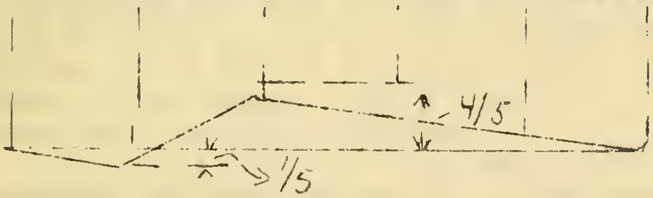
Cross section



Influence line. Shear in Panel 1.

P_o, P_1, P_2, etc. are floor beam reactions

These vary in magnitude for different positions of loads.



Influence line. Shear in panel 2.

Fig. 47.

Consider first the position of loads for shear in panel (1). In this case it is clear that the maximum shear occurs with the same condition that produces maximum moment at panel point a, since the proof given in Article 46 applies equally well here. In consequence one of the loads must lie at a. To determine which, either the method of moving up the loads explained in article 45 may be used, or that of obtaining the position of the loads for maximum moment at a. If the latter

plan be adopted it may happen that more than one position will be found to give a maximum and hence an extra computation will be needed. This latter, is however, useful as a check and is no argument against the method since an approximate check computation with another load at the section should invariably be made.

The fact that the maximum shear in the end panel and the maximum moment at the first panel point occur simultaneously is important. It follows that since none of the live loads can be applied to the girder between panel point, the maximum live moment at the first panel point equals the product of the maximum live shear in the end panel and the length of that panel.

For intermediate panels the latter method can not be used since it is incorrect except for cases where the influence line is composed of two straight lines forming a triangle with the axis of the beam for intermediate panels this condition does not occur. For this case therefore, the method of Article 45 will be adopted. Examination of the influence line in Fig 47, which is typical of the influence lines for all intermediate panels, shows that the loads when brought on from the right must at least be moved to the left until the first load reaches b. Further movement to the left causes a decrease in the shear due to the first load, but an increase due to loads on the right. The result is a net increase. The loads should be moved until load 2 reaches b. This conclusion is uninfluenced by the action of another load coming on the span from the right or by load 1 passing a. Further movement to the left produces an additional increase in shear due to loads to the right of b, but a decrease due to load 2, and either an increase or a decrease due to load 1. If the expression for the change is positive it will remain so until load 3 reaches the section unless load 1 passes off the bridge which would lower the rate of increase and perhaps cause a decrease. This condition is, however, not likely to occur and may be neglected.

In view of the foregoing it may be stated that for maximum positive shear in either end or intermediate panels, one of the loads must lie at the panel point to the right.

Before proceeding to a numerical illustration of these principles the student should observe that the increase in shear in panel 1 equals the increase in R_1 minus the increase in Po, that the increase in shear in panel 2 equals the increase in R, minus the combined increase in Po and P_1, and similarly for other panels.

A load which passes off the span in the process of moving up should always be considered by itself. It should be noted that the change of shear, or of any other function, due to removing a load from a structure, is equal to the shear, or other function, caused by the load when on the structure. Hence to find the change in shear due to a load passing off the span compute the shear due to it in its position at the beginning of the movement, which removes it from the span.

place be assigned to [illegible]

The [illegible]

The [illegible]

In [illegible]

[illegible]

The application of these principles will now be made for the structure shown in Fig. 47 and the locomotive given in Article 45.

Shear in End Panel. Method of Moving-Up the Loads

Start with load 1 at panel point a.

	Increase in R_1		Increase in Po.
Move load 2 to a	$146 \times \frac{8}{60} + \delta$	$>$	$10 \times \frac{8}{12}$

∴ shear is increased. $\delta = 0$ in this case.

Move load 3 to a	$136 \times \frac{5}{60} + \delta$	$>$	$20 \times \frac{5}{12} + 10 \times \frac{4}{12} \times \frac{4}{5}$ *

∴ shear is increased. $\delta = 0$ in this case.

The fact that the increase in moving up P_3 is very slight and that the next step of moving up P_4 would materially increase the change in Po without increasing that in P_1 makes it evident that load 3 at section gives maximum shear.

The value of the maximum shear in the end panel may now be computed. The expression for it is

$$\frac{20}{60}(53+48+43+38) + \frac{14}{60}(29+24+19+14) - 20 \times \frac{5}{12} = 73000 \text{ lbs.}$$

To show the correction of above conclusions the shear with load 2 at a will now be computed.

$$\frac{20}{60}(48+43+38+33) + \frac{14}{60}(24+19+14+9) + 10 \times 4/12 \times 4/5 = 72070 \text{ lbs.}$$

the value of which is less than that for load 3 at section in accordance with the conclusions of the previous method.

* The last term in the above expression gives the shear due to load 1 when load 2 is at a. Its value is obtained by computing the floor beam reaction P_1 and the shear due to it. The reaction Po may be ignored since it produces no shear in the girder. The same result should be obtained by the usual method, of computing R_1 and subtracting Po from it; this gives $\left(\frac{56}{60} - \frac{8}{12}\right) 10$ which equals the value already found.

Shear in End Panel. Average Load Method.

	Av. load per ft. on right		Av. load per ft. on left	
Load 2 to right of panel pt. a	$\frac{136}{4}$	$>$	$\frac{10}{1}$	∴ Load 2 gives a max
Load 2 to left of panel pt. a	$\frac{116}{4}$	$<$	$\frac{30}{1}$	
Load 3 to right of panel pt. a	$\frac{116}{4}$	$>$	$\frac{20}{1}$	
Load 3 to left of panel pt. a	$\frac{96}{4}$	$<$	$\frac{40}{1}$	∴ load 3 gives a max.
Load 4 to right of panel pt.a	$\frac{96}{4}$	$<$	$\frac{40}{1}$	∴ load 4 does not give a maximum

Shear in 2nd. Panel. Method of Moving Up the Loads.

From these expressions it is seen that by the application of the average load criterion, loads 2 and 3 are found to give maxima, and that it is necessary to calculate both to determine the greater.

It should be noted that in the application of the average load method the average shear per panel instead of the average shear per foot has been used. This is simpler and gives the same result when the panels are of equal length as in the bridge under consideration. If the panels are of unequal length this method would be incorrect.

Start with load 1 at panel point b.

	Increase in R_1		Increase in $(P_0 + P_1)$	
Move load 2 to b	$104 \times \frac{8}{60} + \partial$	$>$	$10 \times \frac{8}{10}$	∴ shear is increased

$\partial = 14 \times \frac{4}{60}$ but this value need not have been computed since it will not alter the result.

	Increase in R_1		Increase in $(P_0 + P_1)$	
Move load 3 to b	$132 \times \frac{5}{60} + \partial$	$<$	$10 \times \frac{4}{12} + 20 \times \frac{5}{12}$	∴ 2 gives max.

$\partial = 14 \times \frac{2}{60}$ (necessary to compute in this case since otherwise results would be doubtful.

The right hand side of above inequality may require some explanation. With load 2 at b $P_o = 0$ and $P_1 = \frac{8}{12}\ 10$

With load 3 at b $P_o = \frac{1}{12}\ 10$ and $P_1 = \frac{11}{12}\ 10 + \frac{5}{12}\ 20$

$\therefore$ Increase in $(P_o + P1) = 10 \times \left(\frac{12}{12} - \frac{8}{12}\right) + \frac{5}{12}\ 20$

That is, the increase in $(P_o + P_1)$ when load 1 is moved from the second into the first panel, equals the reaction on the floor beam at b due to load 1 when load (2) is at b plus the increase in the reaction on this floor beam due to moving the second load into the second panel.

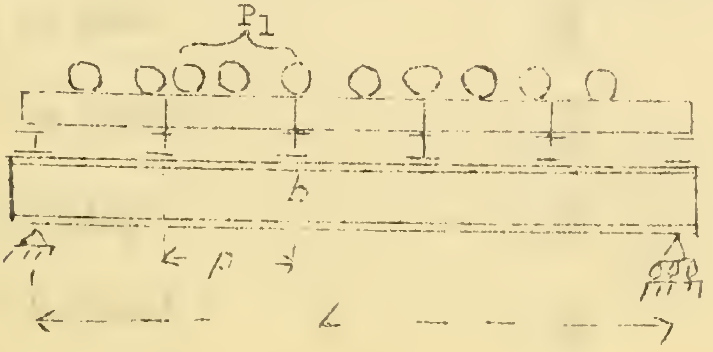
The value of the maximum shear in the second panel equals

$$10 \times \frac{44}{60} + \frac{20}{60}(36+31+26+21) + \frac{14}{60}(12+7+2) - \frac{10 \times 8}{12} = 43560 \text{ lbs.}$$

as an approximate check the shear with load 3 at b has been computed and found to be 43370 lbs. If the increase in shear as determined from the expression for the increase due to moving up load 3 be added to this the result should equal the shear with load 2 at b thus giving an exact check.

48. SHEAR-GIRDER WITH FLOOR BEAMS, ALTERNATE METHOD.

Another method widely used for the determination of the position of the loads for maximum shear in any panel of a girder or truss with floor beams is as follows:-



Let the problem be to determine the position of loads for maximum shear in second panel.

As before one load must lie at panel point b.
Let P = sum of loads on the bridge.
" P_1 = sum of loads in the panel including the load at b

" d_v = increase in shear in given panel due to moving the loads a distance d_x to left.

Then $d_v = \frac{P\,d_x}{L} - \frac{P_1\,d_x}{p}$

$$\therefore \frac{dv}{dx} = \frac{P}{L} - \frac{P_1}{p}$$

In order that the position of the loads may be such as will give a maximum shear in the panel $\frac{dv}{dx}$ must equal or be less than zero.

$$\therefore \frac{P}{L} - \frac{P_1}{p} \leq 0 \qquad \text{or} \qquad \frac{P_1}{p} \geq \frac{P}{L}$$

The application of this method to a given case is as follows: Determine a load such that the average load per foot in the given panel is less than that on the entire span if the load is placed just to the right of the panel, and greater or equal to the average load on the span when placed just in the panel.

This method when applied to the numerical examples of the previous article gives the following results.

Shear in 2nd panel	Average load 2nd panel		Average load entire span	
Load 1 to right	0	<	$\frac{146}{60}$	not a max
" 1 in panel	$\frac{10}{12}$	<	$\frac{146}{60}$	
" 2 to right	$\frac{10}{12}$	<	$\frac{146}{60}$	not a max
" 2 to left	$\frac{30}{12}$	>	$\frac{146}{60}$	
" 3 to right	$\frac{20}{12}$	<	$\frac{136}{60}$	a max
" 3 to left	$\frac{40}{12}$	>	$\frac{136}{60}$	

It is obviously unnecessary to write numerical values for the average loads with load 1 to the right of the panel. It is also clear that for the ordinary case of equal panels the total load on the panel may be compared with the total on the span divided by the number of panels. These simplifications are adopted in determining the positions for maximum shear in the second panel.

Shear in 2nd panel	Total load 2nd panel		Total load on span ÷ number of panels.	
Load 1 in panel	10	$<$	$\frac{104}{5}$	not a max
" 2 to right	10	$<$	$\frac{132}{5}$	a max
" 2 to left	30	$>$	$\frac{132}{5}$	

Unlike the method for moments it is seldom, if ever necessary for shear to try positions after a maximum is once obtained. For ordinary locomotive loadings this need never be done.

This method is simpler to use than the method of moving up the loads.

The student should, however, thoroughly familiarize himself with the latter method in order that he may gain a clear insight into the influence of the loads upon the structure.

49 MAXIMUM MOMENT. GIRDER with FLOOR BEAMS. For girders with floor beams it is customary to compute maximum moments at panel points only. If, for any reason, the maximum moment between panel points is desired it may be obtained with sufficient accuracy by interpolation.

For uniform live loads and for concentrated loads which are fixed in position interpolation gives exact results since the curve of moments for such loads consists of a series of straight lines. The same is also true for moments due to weight of the floor system, but is slightly in error for the weight of the girder itself. For a system of moving loads this method is somewhat inaccurate but is on the safe side and hence may be used with security. This is shown by the following demonstration.

Let the ordinate b A represent the maximum live moment at b due to a concentrated load system. For the position producing this maximum the moment curve for the portion of the girder between b and c will be the line A B, B c representing the moment at c for the position of the loads giving maximum moment at b. If the loads be now moved so as to give the maximum moment at c we shall have c B' and b A' as the ordinates for momtnts at c and b respectively for this new position and B' A' will be the moment curve between b and c. It is evident from the figure that interpolation between the maximum moment at b and that at c will give a safe value for the maximum moment at any point in the panel, since the line A B can never rise above A B' nor the line B'A' above B'A, therefore, the ordinate XX' for the moment at X can never be less than the actual maximum moment at X. It will readily be seen by drawing an influence line for the moment at X that for maximum moment some load should lie at either panel point b or c. that the moment at c with the loads in the position necessary for max. moment at b can never exceed the maximum moment at c and will almost invariably be less than that, and that this principle holds good for the condition when the moment at b is a maximum. This proof is perfectly general and applicable to any panel.

50. MOMENT and SHEAR at CRITICAL SECTION. The cases already treated have been for shear and moment at stated sections of simple beams and for panels and panel points of girders with floor beams. For the latter it is necessary and sufficient to compute shear in each panel and moment at each panel point, since thereby the maximum values of these functions will be obtained. For beams or girders which are not loaded by floor beams it is always necessary to compute maximum values of shears and moments, and in addition, for long girders, the values at intermediate sections taken with sufficient frequency to ensure a good design.

In order to determine the maximum values it is necessary to locate the sections at which they occur, that is the critical sections.

For shear the critical section is an infinitesimal distance from one of the points of support. This needs no demonstration, as an inspection of influence lines for various sections including one at the end furnishes sufficient proof.

For moment with uniform load it has already been shown that the maximum moment occurs at the centre and equals $\frac{1}{8} WL^2$

when w equals the load per foot and L the span.

With a system of concentrated loads the maximum moment may not occur at the centre though the critical section will be very near the centre. To treat this case it is necessary to make use of the already established principle that for maximum moment at any section of a beam under a system of concentrated loads one of the loads must lie at the section. If, therefore, it is possible to determine the location of the system of loads as they cross the span such that the moment at any one load is a maximum the problem can be solved

by trying a sufficient number of loads and computing the different maxima. As will appear later the critical section is always near the centre of the span hence, as a rule, those loads only need be tried which are found to give a maximum moment at the centre.

Consider the set of loads shown below and let the problem be the determination of the position of these loads in order that the moment at P_3 may be a maximum. Let R be the resultant of the loads P_1 to P_5 inclusive and n its distance from the last load P_5.

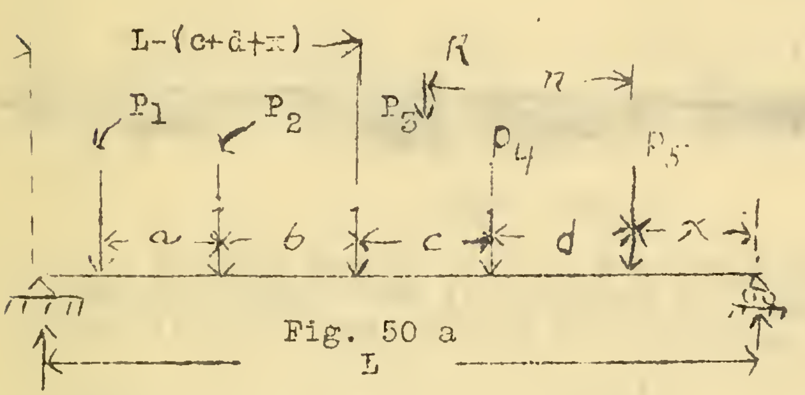


Fig. 50 a

Let x be the distance from P_5 to the right support when the loads lie in the proper position for maximum moment at P_5.

$$\frac{R(n+x)}{L} = \text{reaction}$$

Then the moment at P_3 is given by the equation.

$$M_3 = R\,\frac{(n+x)}{L}\left[L - (c+d+x)\right] - P_1\,(a+b) - P_2\,b$$

For max. value of M_3 differentiate with respect to x and put the first derivative equal to 0. This gives

$$\frac{dM_3}{dx} = \frac{R}{L}\left[-n + L - c - d - 2x\right] = 0.$$

Therefore, in order to find the maximum moment at P_3 as the loads cross the span P_3 must be so located that

$$-n + L - c - d - 2x = 0$$

or

$$L - (c+d+x) = n + x.$$

That is, the resultant of the loads on the span when the maximum moment at P_3 occurs must lie as far from the right support as the load itself lies from the left support, or in other words the centre of the span must lie half way between the resultant and the load.

[illegible]

[illegible]

[illegible]

Then the moment [illegible] is given by the equation,

[illegible]

[illegible]

[illegible]

Therefore, in order to find the maximum moment [illegible]

[illegible]

[illegible]

The following examples serve to illustrate the application of this principle;-

Prob. 506. Compute absolute maximum moment due to loads shown on beam in Fig. 506

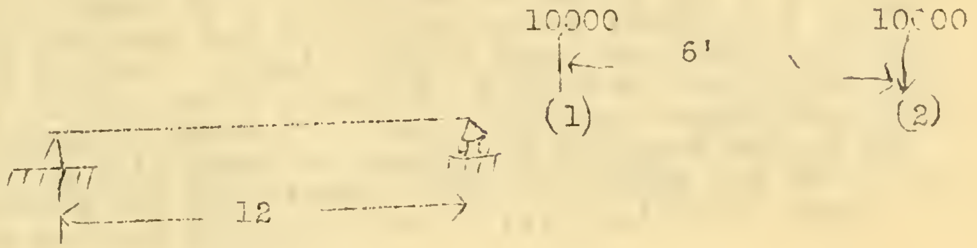


Fig. 506.

In this case there are two equal loads, hence it is immaterial which load is considered. For maximum moment at load (1) the loads should lie as shown in Fig. 50 c

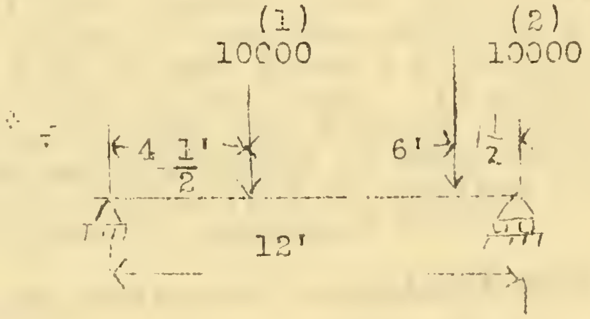


Fig. 50c

the centre of the span being half way between load (1) and the resultant of the two loads. The moment at the first load will then be

$$20000\frac{(6-1\frac{1'}{2})^2}{12} \text{ or } 20000\frac{(6+1\frac{1}{2})^2}{12} - 10000 \times 6$$

= 33750 ft. lbs. The maximum moment at the centre for this beam would be 30000 ft, lbs., hence the absolute maximum moment exceeds the maximum centre moment by over 10 %.

It should be particularly noted that the demonstration which has been given only serves to fix the position for maximum moment at a given load with certain assumed loads on the span, and that if a different set of loads be on the span the position will be different. To illustrate this consider the same loads in the previous examples and a span of 10'. There are then two positions of the first load which give maximum moment. First, assume only the first load on the span; in this case it should be placed at the centre and the moment would be 25000 ft. lbs. Second, assume two loads on the span; in this case the centre of the span should be half way between the resultant of the two loads and the first load and the maximum moment at the first load will equal

$$20000\frac{(5 - 1\frac{1}{2})^2}{10} = 24500 \text{ ft. lbs.}$$

which is somewhat less than with one load at the centre. In such a case the length of span can easily be determined at which one load at the centre gives a moment at the load equal to that with two loads on the span. In general it is necessary to consider both cases when dealing with two loads.

The absolute maximum moment on spans above 25' or 30' doessnot materially differ from the maximum centre moment and in practice the latter is generally used.

For the loads previously considered with a 30' span the absolute maximum moment $= 20000\frac{(15 - 1\frac{1}{2})^2}{30} = 121500$ ft. lbs. while maximum centre moment $= 20000\left(\frac{12}{30}\right) 15 = 120000$ ft. lbs. the difference being about 1 percent, this being so small as to be negligible.

The following examples serve to show the application of this principle for the locomotive loading given in Article 45.

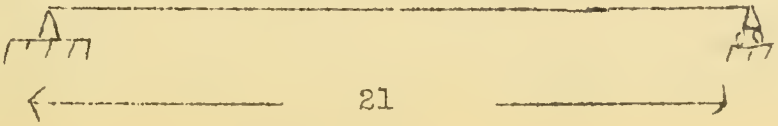


Fig. 50d

First determine which load or loads give maximum moment at the centre, as it is probable that one of these loads will give the absolute maximum moment. By applying the criterion for maximum moment loads 3 and 4 are found to give maxima, but it is clear that the centre moment with load 3 at the centre will equal the centre moment with load 4 at the centre, and that it makes no difference whether we use one or the other loads. Let the maximum moment therefore be determined at load 3, assuming loads 2 - 5 inclusive on the span. The position for maximum moment will then be as shown in Fig.50c and the moment at load (3) will equal

$$\frac{80000\,(10\tfrac{1}{2} - 1\tfrac{1}{4})^2}{21} - 20000 \times 5 =$$

225950 ft. lbs.

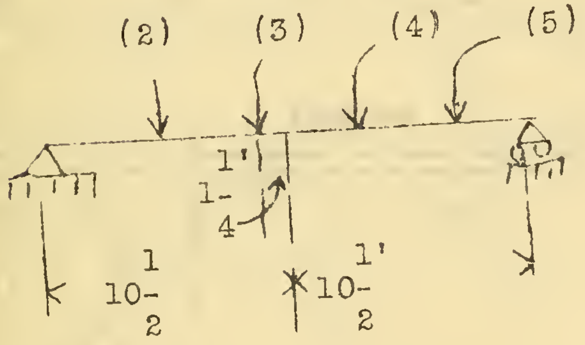


Fig. 50c.

In this case it is impossible to get more than four loads on the span at once. If three loads are on the span the resultant coincides with load 3 hence, for a maximum in this assumption, load 3 should lie at the centre but this is inconsistent with three loads being on the span, hence a maximum at load 3 with only three loads on the span cannot be obtained and the case considered gives the absolute maximum moment.

The maximum centre moment for these loads occurs with either load (3) or load (4) at the centre and equals -

$$80000 \times \frac{8}{21} \times 10\frac{1}{2} - 20000 \times 5 = 220000 \text{ ft. lbs.}$$

so that in this case the difference is only 2.7%.

51 MOMENTS and SHEARS, FLOR BEAMS and TRANSVERSE GIRDERS. As a preliminary step in the examination of this case the influence lines shown in Figs. 51a and 51b have been drawn. These are influence lines for stringers reactions on floor-beams. Since the stringers are simple beams of a length equal to one panel and are supported at the ends upon the floor-beams. It is evident that a load moving along the bridge caused no reaction on a floor beam unless it is on the stringers in one of the panels adjoining the floor beams in question. Fig. 51a represents the stringer reactions on an intermediate floor-beam and 516 on an end floor-beam.

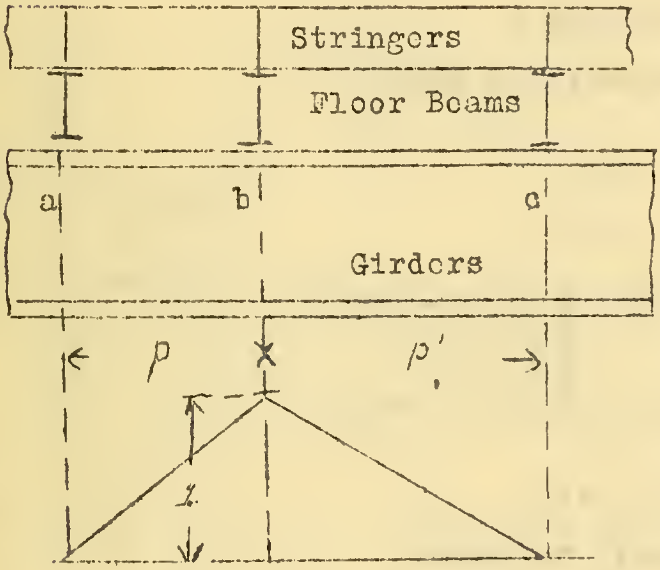


Fig. 51a

Influence Line. Reaction on floor-beam at b

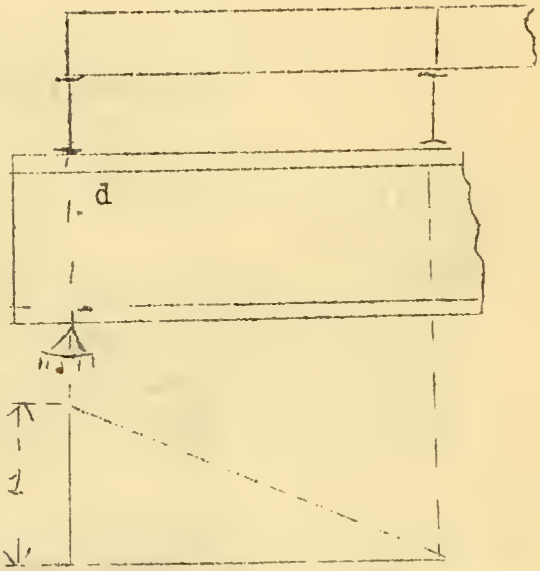


Fig. 51b

Influence Line. Reaction on floor-beam at d. (End floor-beam

It will be noticed that the influence line shown in Fig. 51a has the same characteristics as the influence line for moment at any section of a simple end-supported beam, hence the demonstration of Article 46 is applicable. The conclusion may therefore be at once drawn that for maximum reaction on an intermediate floor-beam one load must lie at the beam, and that load must be one which when placed just to the right of the given floor-beam makes the average load per foot on the stringers in the right hand panel greater than on those in the left panel, and when placed just to the left of the floor moment at [illegible]. From [illegible] two different diagrams, as a preliminary step [illegible] the [illegible] of influence lines [illegible] Figs. 17a and 17b [illegible]. [illegible] influence lines [illegible] drawn [illegible] the [illegible] of a [illegible] for the [illegible] and [illegible] in the same way [illegible]. It is [illegible] that a load moving along [illegible] floor [illegible] the two [illegible] at the panel points [illegible] the floor beams [illegible]. Fig. 17a [illegible] intermediate floor-beams and [illegible] diagonals.

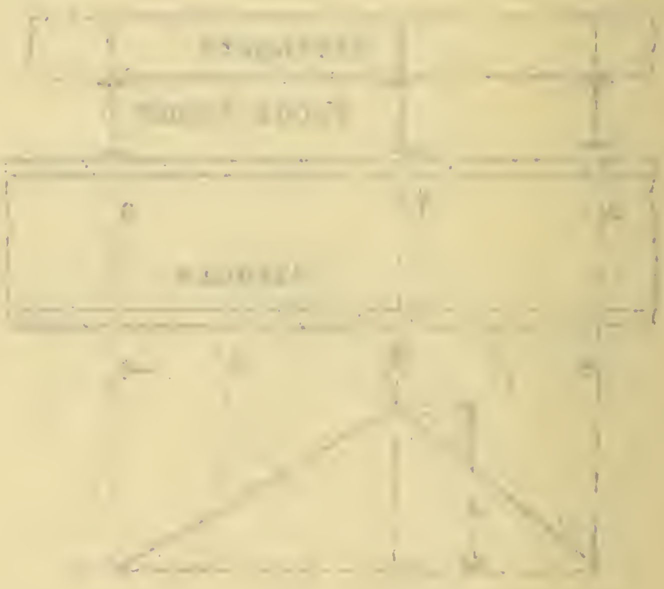


Fig. 17a

Influence line. Reaction at right end of A.

Fig. 17b

Influence line. Moment on floor-beam at right of panel-ing

It will be noticed that the influence line shown in Fig. 17b [illegible] the two ordinates [illegible] the [illegible] floor [illegible]. The conclusion [illegible] intermediate floor [illegible] the [illegible] panel [illegible] the floor beams [illegible] the right [illegible] half of the floor [illegible] beam reverses this condition. For the end floor-beam, the maximum reaction occurs under the loading giving maximum stringer reaction and equals that reaction.

It remains to consider the actual moments and shears on the floor-beams. Curves of moments and shears for a floor beam due to stringer reactions are shown in Figs. 51c and 51d.

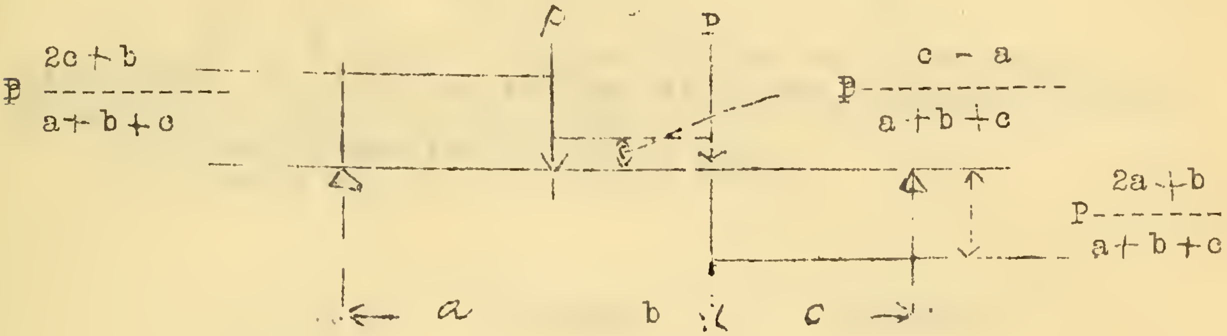


Fig. 51c.

Curve of floor-beam shear.

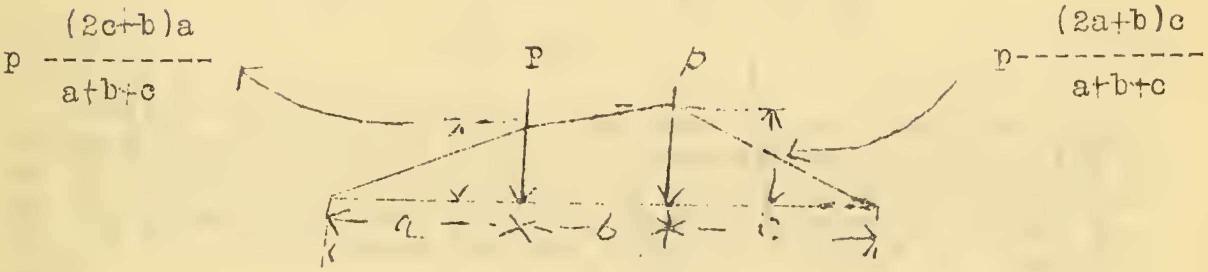


Fig. 51d

Curve of floor-beam moments.

Both shear and moment are direct functions of the stringer reactions. The maximum moment must occur at one of the stringers, since the floor beam is in the condition of a girder loaded with concentrated loads and the curve of shears can cross the axis only at a load. The case illustrated is an unusual one since the stringers are unsymmetrically placed with respect to the centre of the floor beam. Were the floor beam symmetrical the maximum moment would exist at both stringer and at all points between. Since in the actual design the dead load of the floor beam would also have to be considered the maximum combined live and dead moment for the ordinary symmetrical floor beam occurs at the centre.

For floor beams where the stringers in one of the adjoining panels are not of equal length, that is where the panel is a skew panel, special treatment is necessary. It is usually advisable to treat this case by means of influence lines without the attempt to apply special rules.

The application of the methods just given to the determination of the maximum moment and shear on a floor beam (or a transverse girder such as a cross girder in an elevated railroad structure) will now be illustrated.

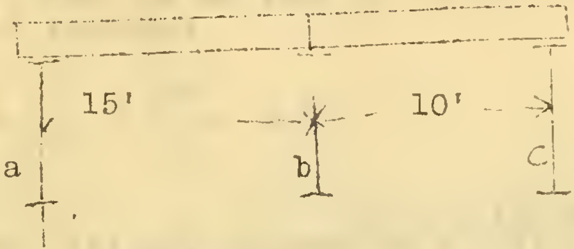


Fig. 51e.

The floor beam b in Fig 51e is the one to be figured. Its length, the distance between stringers, and the location of girders is shown in Fig. 51f.

The load system to be used is that of Article 45.

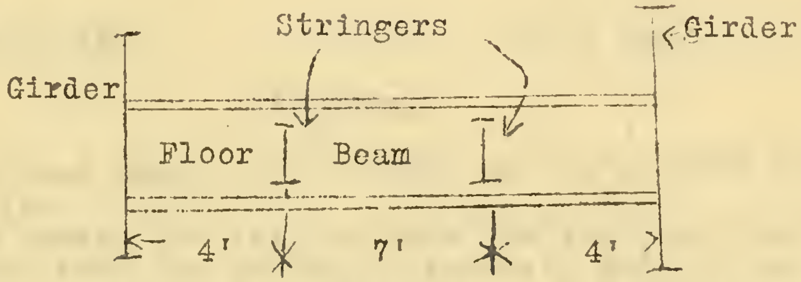


Fig. 51f.

Since it may be easily seen that given load when equidistant from the floor-beam, b, produces greater reaction if on the longer span it is probable that the maximum reaction in this case will come with the greater number of loads on the 15 ft. panel hence let the loads be brought on from the left

	Av. load per ft. on right.		Av. load per ft. on left.	
Load 2 to left	$\frac{10}{10}$	<	$\frac{60}{15}$	load 2 does not give a max.
2 to right	$\frac{30}{10}$	<	$\frac{60}{15}$	
3 to left	$\frac{20}{10}$	<	$\frac{60}{15}$	load 3 gives a max
3 to right	$\frac{40}{10}$	>	$\frac{40}{15}$	
4 to left	$\frac{40}{10}$	>	$\frac{54}{15}$	load 4 does not give a max.

Load 3 at b gives the maximum floor beam reaction. Its value is given by the expression.

$$20 \times \frac{5+10}{10} + 20 \quad \frac{10+5}{15} = 50$$

That is, the reaction on the floor beam at each stringer connection equals 50000 lbs.

The floor beam is then in the condition shown by Fig. 51g.

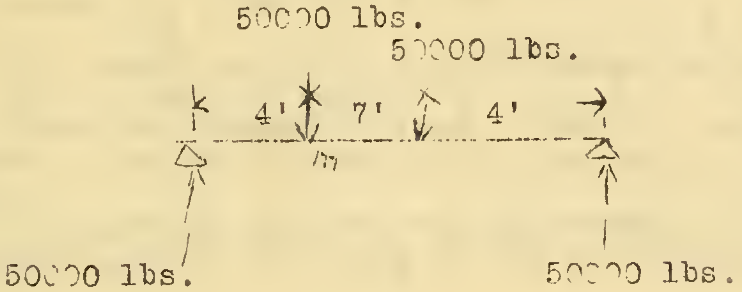


Fig. 51g.

The maximum shear = 50000 lbs and the maximum moment = 200000 ft. lbs.

Before concluding this article the beginner should be cautioned to avoid the mistake frequently made of doubling the maximum reaction on one stringer to determine the floor beam load at a point such as M in Fig. 51g. The fact that the maximum reaction on a stringer occurs when one of the heavy loads lies at the end of the stringer is sufficient to show that the same conditions cannot exist on the next stringer because such a condition would necessitate having two wheel loads occupying practically the same place at the same time.

MOMENT DIAGRAM. To save repetition of computations for a given set of concentrated loads when used for varying spans it is customary to employ a moment diagram upon which certain quantities frequently required and unaffected by the length of spans are placed once for all. Upon this diagram the loads are plotted to a convenient scale at top and bottom of sheet for convenience in reading, and the quantities desired are placed between. The diagram is used in connection with another sheet upon which the span is drawn to the scale used in plotting the loads. The diagram shown in Fig. 51a, is of a convenient form for use though less comprehensive than some, and is self-explanatory.

The use of the diagram can be readily understood from the simple example that follows. Let the problem be to compute the moment at the second panel point from the left of a span having 5 panels @ 20' when load 8 is at the given panel point. Place the plotted span on its separate sheet so that load 8 is over panel point 2; the ends of the span will then be in the positions shown at bottom of diagrams. Since the desired moment equals the moment of the left reaction due to loads 2 to

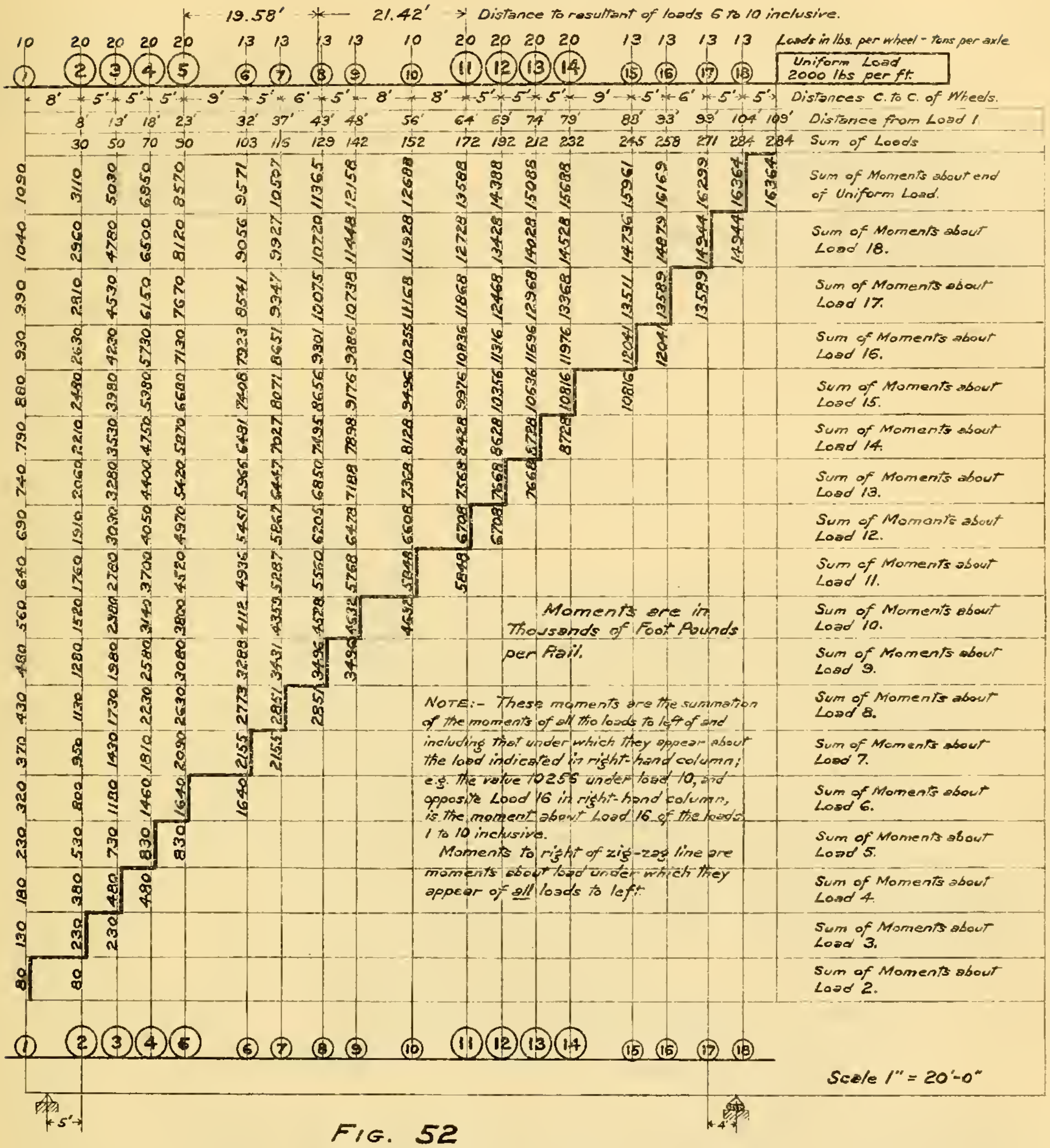
17 inclusive about load 8, minus the moment of loads 2 to 7 inclusive about the same point, it is necessary to compute these two quantities. The moment of the left reaction equals the moment of the loads on the span about the right reaction divided by 5 panel lengths and multiplied by 2 panel lengths. This equals $\frac{2}{5}\ 13589-990+(271-10)x4 = \frac{2}{5}\ 13643 = 5457.2$

The moment of loads 2 to 7 inclusive about panel point 2 = 2851 - 10x43 = 2421.

Hence the moment desired = 5457.2 - 2421 = 3036.2 expressed in units of thousands of pounds per rail or tons per track.

This method involves the application of the principle stated in article 34 which should be thoroughly understood.

MOMENT DIAGRAM — COOPER'S E 40 LOCOMOTIVE.


19.58'
21.42'
Distance to resultant of loads 6 to 10 inclusive.
Uniform Load 2000 lbs per ft.
Distances c. to c. of Wheels: 8' 5' 5' 5' 9' 5' 6' 5' 8' 8' 5' 5' 5' 9' 5' 6' 5' 5'
Scale 1" = 20'-0"
5'
4'

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
Loads in lbs. per wheel - Tons per axle	10	20	20	20	20	13	13	13	13	10	20	20	20	20	13	13	13	13	
Distance from Load 1		8'	13'	18'	23'	32'	37'	43'	48'	56'	64'	69'	74'	79'	88'	33'	99'	104'	109'
Sum of Loads		30	50	70	90	103	115	129	142	152	172	192	212	232	245	258	271	284	284
Sum of Moments about end of Uniform Load.	1090	3110	5030	6950	8570	9571	10507	11365	12158	12698	13588	14388	15088	15688	15961	16169	16299	16364	16364
Sum of Moments about Load 18.	1040	2960	4780	6500	8120	9056	9927	10720	11448	11928	12728	13428	14028	14528	14736	14879	14944	14944	
Sum of Moments about Load 17.	990	2810	4530	6150	7670	8541	9347	10075	10738	11168	11868	12468	12968	13368	13511	13589	13589		
Sum of Moments about Load 16.	930	2630	4230	5730	7130	7923	8651	9301	9886	10256	10836	11316	11696	11976	12041	12041			
Sum of Moments about Load 15.	880	2480	3980	5380	6680	7408	8071	8656	9176	9496	9976	10356	10636	10816	10816				
Sum of Moments about Load 14.	790	2210	3530	4750	5870	6481	7027	7495	7898	8128	8428	8628	8728	8728					
Sum of Moments about Load 13.	740	2060	3280	4400	5420	5966	6447	6850	7188	7368	7568	7668	7668						
Sum of Moments about Load 12.	690	1910	3030	4050	4970	5451	5862	6205	6478	6608	6708	6708							
Sum of Moments about Load 11.	640	1760	2780	3700	4520	4936	5287	5560	5768	5848	5848								
Sum of Moments about Load 10.	560	1520	2380	3140	3800	4112	4351	4528	4632	4632									
Sum of Moments about Load 9.	480	1280	1980	2580	3080	3288	3431	3496	3496										
Sum of Moments about Load 8.	430	1130	1730	2230	2630	2773	2851	2851											
Sum of Moments about Load 7.	370	950	1430	1810	2090	2155	2155												
Sum of Moments about Load 6.	320	800	1180	1460	1640	1640													
Sum of Moments about Load 5.	230	530	730	830	830														
Sum of Moments about Load 4.	180	380	480	480															
Sum of Moments about Load 3.	130	230	230																
Sum of Moments about Load 2.	80	80																	

Moments are in Thousands of Foot Pounds per Rail.

NOTE:- These moments are the summation of the moments of all the loads to left of and including that under which they appear about the load indicated in right-hand column; e.g. the value 10256 under load 10, and opposite Load 16 in right-hand column, is the moment about Load 16 of the loads 1 to 10 inclusive.

Moments to right of zig-zag line are moments about load under which they appear of all loads to left.

FIG. 52

Application of Diagram. To find moment about a given load, say load 14 of a certain number of loads to left, say 6 to 13 inclusive, proceed as follows:- Find in diagram, moment about 14 of all loads to left; subtract from it moment about 5 of all loads to left of 5 plus the product of the sum of loads 1 to 5 inclusive and distance from load 5 to load 14. The result will be the desired moment. The expression thus obtained = 8728 − [830 + 90 × 56] = 2858 The moment of the same loads about a point between 14 and 15 and distant X ft. from 14 can be found by adding to quantity just obtained the product of the sum of the loads 6 to 13 and the distance X. See Article 34.

CHAPTER IV.

DESIGN OF BEAMS AND GIRDERS.

53. BEAM FORMULAS. In order to determine the proper size of beams to carry given external bending moments and shears it is necessary to make use of formulas expressing the relation between the outer and inner forces. Such formulas are reduced in all standard books on Mechanics and are as follows for beams of honogeneous material of ordinary proportions.

$$M = \frac{f\,I}{V} \qquad 53a$$

and $$V = \frac{S\,Q}{b\,I} \qquad 53b$$

In these formulas

M = maximum allowable external bending moment in inch lbs. at a given cross section of the beam.
S = maximum allowable external shear in lbs. at a given cross section.
I = moment of inertia in inches 4 of given cross section about its neutral axis.
y = distance in inches from neutral axis to extreme fibre of the cross section.
Q = statical moment about the neutral axis of that portion of the cross section lying between the neutral axis and the outer fibre.
v = allowable intensity of the longitudinal shear per square inch.
f = allowable direct stress (in bending) per square inch in extreme fibre.
b = width of beam at neutral axis.

By properly applying these formulas it is possible to design beams for which the maximum moments and shears are known.

54 BEAM DESIGN. Frequently the design of beams requires merely the application of the formula $M = \frac{f\,I}{y}$, and the shearing strength need not be considered. In the case of comparatively short beams, however, the shearing resistance becomes more important and should be investigated. In wooden beams this is especially important, since the resistance of wood to longitudinal shear is small and such beams may fail by splitting

longitudinally. As wooden beams are usually rectangular in cross section the formula $M = \frac{f\,I}{y}$ may, for a rectangular beam, be put into the form $M = 1/6\ f\ b\ h^2$ in which b = width of beam in inches, h = height in inches, and f = allowable unit stress in extreme fibre.

The formula $V = \frac{SQ}{b\,I}$ may also, for rectangular beams, be put into the following simple form; $V = \frac{3\,S}{2hb} = \frac{3S}{2A}$ where A = area of cross section in square inches.

55 WOODEN BEAMS. In selecting wooden beams care should be taken to use commercial sizes only. The following table gives such sizes;-

Spruce
2 X 3, 2X4, 2X5, 2X6, 2X7, 2X8, 2X9, 2X10, 2X12.
3 X 4, 3X6, 3X7, 3X8, 3X9, 3X10, 3X12.
4 X 4, 4X6, 4X8, 4X9, 4X10, 4X12.
6 X 6, 6X8, 6X10, 6X12.
8 X 8, 8X10, 8X12.
10 X 10, 10 X 12.

12 ft. to 22 ft. length are ordinary lengths.
23 ft. to 26 ft. " " less common.
27 ft. to 32 ft. " " obtained with difficulty.

Yellow Pine.
Sizes about the same as for spruce, also
2 X 14, 2 X 16, 2 X 18.
3 X 14, 3 X 16, 3 X 18.
4 X 14, 4 X 16, 4 X 18.
6 X 14, 6 X 16, 6 X 18.
8 X 14, 8 X 16, 8 X 18.
10 X 14, 10 X 16, 10 X 18.
12 X 14, 12 X 16, 12 X 18.
14 X 14, 14 X 16, 14 X 18.
16 X 16, 16 X 18.
18 X 18.

Lengths are longer than for spruce and run up to 40 ft. and it is usually possible to obtain even 50 ft. to 60 ft. lengths except for the largest sizes.

The cost of wooden beams depends upon the price of lumber per board foot. This is subject to considerable variation, and if a close estimate is desired, a dealer should be consulted.

56 STEEL BEAMS. These beams are always made in an I shaped form in order to obtain a large moment of inertia from a comparatively small amount of material. They are rolled from a solid piece of steel in varying heights and thicknesses.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Yellow wood.

[illegible]

[illegible]

[illegible]

[illegible]

In selecting such beams the manufacturers' handbooks should be consulted, and sections marked "standard" chosen since the selection of a "special" section is likely to cause delay in filling the order. These handbooks give all the properties of the beams such as area, weight, moment of inertia, etc., and may be relied upon as accurate.

The cost of steel beams is dependent upon the weight of the beam, and upon the amount of punching, riveting and other work which has to be done. The price is usually figured on a "cent per pound" basis, the price of the unfabricated beam being taken as the base price, and the other prices being added. Other things being equal the lightest beam with the requisite strength and stiffness is most economical. The base price is published from time to time in such engineering papers as the "Engineering News", "Iron Age," etc., e.g. in the Engineering News" of Dec. 2, 1909, the f.o.b. Pittsburg price was quoted as 1.55 cents per pound for 3" to 15" I beams and channels and 1.60 cents for depth greater than 15". The freight rates for carload lots from Pittsburg as quoted in this same issue were as follows:- To New York 16 cents and to Boston 18 cents per 100 lbs. This price is for beams cut to length with a variation not to exceed 3/8" more or less than specified. For cutting to more exact length and for other work the following schedule adopted in January 1902 gives the extra cost.

1. For cutting to length with less variation than plus or minus 3/8" .15 per 100 lbs.
2. Plain punching one size hole in web only-0.15 per 100 lbs
3. " " " " " " one or both flanges- 0.15 " 100 "
4. Plain punching one size hole in either web and one flange or web and both flanges- 0.25 " 100 "
5. Plain punching each additional size hole in either web or flange, web and one flange, or web and both flanges - 0.15 per 100 "

57. EXAMPLES OF BEAM DESIGN.

Problem. Design wooden and steel beams for 12ft. span Beams to be supported at ends and to be loaded with a total uniform load of 1000 lbs. per ft. Allowable unit stresses to be those given in Article 19 for steel and yellow pine. 50% to be added for the steel beam to allow for impact.

Maximum moment is at centre of beam and equals

$$\frac{1}{8}\ 1000.\ 12.\ 12 = 18000 \text{ ft. lbs.}$$

Maximum shear is at end of beam and equals 6000 lbs.

For wooden beam $M = \frac{1}{6} f\ b\ h^2 \therefore 18000 \times 12 = \frac{1}{6}\ 1300\ h^2 \therefore bh^2 = 997.$

Either an 8" x 12", 6" x 14" or 4" x 16" beam has a value of bh^2 greater than that required and may be used.

The area of cross section needed to carry shear may be determined from the formula.

$$v = \frac{3S}{2A} \qquad v = 120, \qquad S = 6000 \therefore A = \frac{18000}{240} = 75 \text{ sq. inches}$$

∴ The 4" x 16" beam is too small to shear and one of the other beams should be selected. The 6" x 14" is the cheapest and should be chosen if conditions permit, and if it can be well braced against lateral deflection. If the beam can not be secured laterally the 8" x 12" should be used. The longer side should always be vertical for economy and was so assumed on solving for bh^2. None of the beams selected would be strong enough if laid flat.

The bearing area on the abutment must also be computed. If the reactions were uniformly distributed over the bearing surface there would be needed $\frac{6000}{260}$ = 23 sq. inches. To allow for unequal distribution 50% will be added to this, giving 345 sq. inches. The 6" x 14" beam would therefore need to extend $\frac{34.5"}{6}$ or say 6" over the abutment.

For the steel beam the moment after allowance for impact is 27000 ft.lbs. $\frac{1}{y} = \frac{27000 \times 12}{16000} = 20.25$

The term is known as the section modulus. Value of this for various beams are given in the handbooks issued by different steel makers, and the lightest beam having a modulus equal to or greater than the above figure should be selected. A 10" I beam, weighing 25 lbs. per ft. is amply strong and will be chosen. A 9", 25 lb. beam is also strong enough, but as this is just as expensive as the 10" beam and not so strong the 10" beam should be selected provided conditions do not require the use of a shallower beam.

The distribution of shear over an I beam is more complicated than for a rectangular beam. It will, however be shown later that the shear is practically all carried by the web over which it is distributed almost uniformly. Making this assumption the area required in the web would be $\frac{9000}{12000}$ of a square inch and as the actual area in the beam selected is far in excess of this the beam is evidently strong enough to carry the shear and hence may be used with safety.

Problem. Design wooden and steel beams for span and loads shown in Fig 57. Track weighs 400 lbs. per linear foot per track. Allow 25 percent for impact. Use unit stresses given in Article 19.

wheel loads.

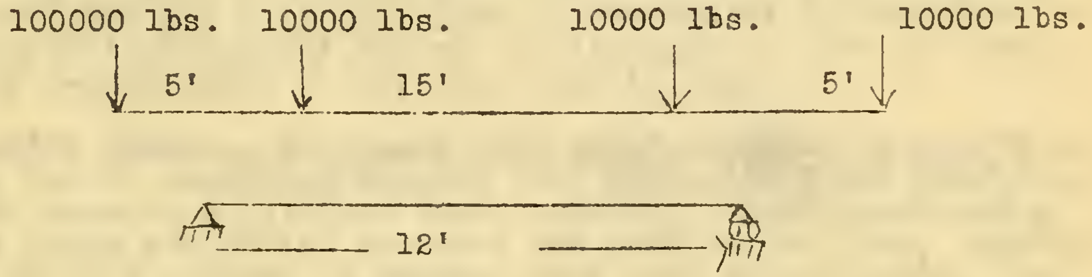


Fig. 57.

Assume dead weight of beam as 40 lbs. per foot,
then total dead load per linear foot for one beam = 240 lbs.

Maximum moment

$$\text{Dead} = \frac{1}{8}\,240.12.12 = 4300 \text{ ft.lbs.}$$

$$\text{Live} = 20000\,\frac{(6-1.25)^2}{12} = 37600$$

$$\text{Impact} = \underline{9400} \text{ "}$$

$$\text{Total moment} = 51300 \text{ "}$$

Maximum shear

$$\text{Dead} \quad 240\text{x}6 = 1440 \text{ lbs.}$$

$$\text{Live} \quad 10000\text{x}\frac{19}{12} = 15830 \text{ "}$$

$$\text{Impact} = 3960 \text{ "}$$

$$\text{Total shear} = 21230 \text{ lbs.}$$

$$\text{For steel beam } \frac{1}{y} = \frac{51300\text{x}12}{16000} = 38.5$$

$$\text{Area needed for shear} = \frac{21230}{12000} = 1.77 \text{ sq. inches.}$$

A 12" I-40 lbs. is large enough for bending and as it has a web area of 6.72 sq. inches its strength in shear is far greater than necessary.

As the actual weight of the beam equals that originally assumed no recomputation is necessary. A considerable error might, however, have been made in the original assumption without requiring a recomputation since the moment and shear due to the weight of beam is a very small percentage of the total moment and shear.

$$\text{For wooden beam neglecting impact } 41700 \text{ x } 12 = \frac{1}{6}\,1300bh^2 \therefore bh^2 = 2310$$

$$\text{Area needed for shear} = \frac{3}{2}\,\frac{17270}{120} = 216 \text{ sq. inches}$$

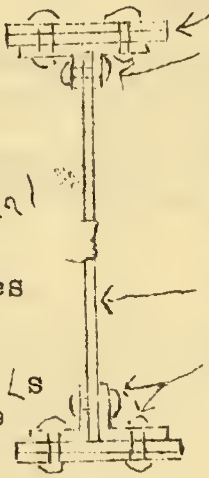
One beam 14"x 16" with 16" side vertical fulfills both requirements and will be chosen. Its weight is somewhat in excess of that assumed but as its strength is also in excess of the requirements no revision need be made.

58. COMPOSITE BEAMS. The cases just treated are of simple beams only, but it sometimes happens that composite beams are used, as for example a flitched beam where two wooden beams and a steel plate are bolted together and used as one beam, Another case is of two beams of unequal size laid side by side. For this case and for the flitched beam, the load carried by each is in proportion to the product of its moment of inertia and modulus of elasticity and can be easily computed. Another case which should be noted is that of one beam laid on the top of another, but not riveted to it. Such a beam is of slightly greater strength than two beams laid side by side; the additional strength is due, however, to friction between the beams and should be neglected in design. If the beams are riveted together with a sufficient number of rivets to carry the longitudinal shear existing at the plane of contact, assuming the beam to be solid they may be figured as one beam of a height and cross section corresponding to the combination. Reebforced concrete beams form the most important class of composite beams. These will be discussed in another portion of this book.

59. STIFFNESS. Beams are seldom used for bridge spans longer than 30' since above that span the ratio of length to depth is so great that the deepest beam made, a 30" I beam, lacks sufficient stiffness. For buildings longer beams are admissible, but the length should generally be restricted to twenty times the depth for floors and less than that if the floor be subjected to vibration and shock. In roofs somewhat longer beams may be used. When the beams are not supported laterally the ratio of length to width of the compression flange should be considered and the allowable unit stresses reduced accordingly. A common rule is to make no reduction in allowable stress when the ratio of unsupported length to width does not exceed 20, to take half this stress when the ratio is 60, and proportionally for intermediate lengths.

60 PLATE GIRDERS DEFINED. For lengths greater than are admissible for steel beams or where beams of sufficient strength cannot be obtained of other types of structures must be adopted. Of these the plate girder is the next higher form.

A plate girder is essentially an I beam made, not out of one solid piece of metal, but out of a number of pieces riveted together. Fig. 60a shows a typical plate girder with the different parts named



Cover plates

Flange or chord ∠s
Cover plates and flange ∠s combined make the flanges. In some girders cover plates are not used

Web plate

Rievts

Depth back to back of ∠s usually made 1/2" nominal greater than depth of web to ensure that edges of web will not project beyond the back of flange ∠s and there interfere with cover plates

Fig. 60a.

Cross-section of a Plate Girder.

Plate girders are rarely made of greater depth than 10'-6" owing to difficulties in transportation by rail, and a length of 100 ft. is seldom exceeded for the same reasons although girders of 125 ft. in length have been made and shipped in one piece. Occasionally plate girders are made in sections and spliced in the field but this expedient is not common and should not be adopted except to meet some unusual condition.

Plate girders are sometimes made with more than one web as indicated in Fig 60b. Such a girder is called a box girder. Its principal use is for situations where great strength with limited depth is required.

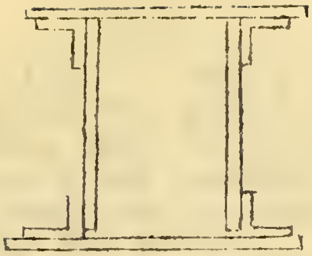


Fig. 60b.

Box Girder.

61 PLATE GIRDER WEB THEORY. Plate girder webs are proportioned on the assumption that all the transverse shear is uniformly distributed over the net area of the web, which may be taken as 3/4 the gross area without material error. That this assumption is essentially correct is shown by the following demonstration:

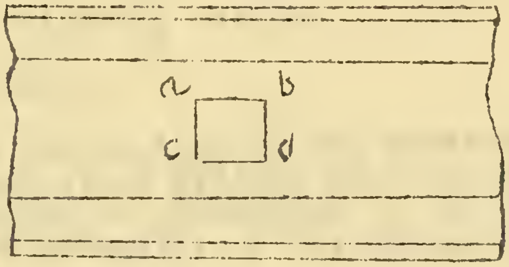
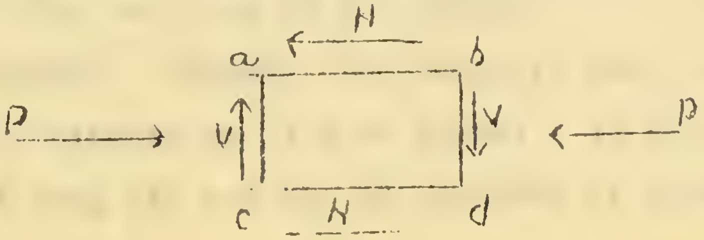


Fig. 61a.

In Fig. 61a the figure abcd represents an elementary prism in the web of a plate girder. Assume this prism to be so small that no appreciable error occurs by assuming as equal the direct stresses on the two opposite faces. The forces acting upon the prism will be the direct fibre stresses on the faces ac and bd, and shearing stresses on all the faces, It is evident that in order to satisfy the equations, $\Sigma V = 0$, and $\Sigma H = 0$, the vertical forces must be equal and opposite in direction, and the same relation must exist between the horizontal forces. This condition is shown in Fig. 61b in which the forces shown are the total forces acting on the respective sides.



In order that ΣM may = 0 the moment of the horizontal forces must equal the moment of the vertical forces, from which it follows that the intensity of the shear at any point upon a vertical plane passing through the point equals the intensity of shear at the same point upon a horizontal plane passing through the point.

This may be demonstrated as follows.

Let t = thickness of web.
" S_h = intensity of shear on the horizontal surface
" S_v = intensity of " " " vertical "
" unity = length of the sides;

Apply the equation $\Sigma M = 0$ about one of the corners.

Then $(S_h\ t)\quad(1) = (S_v\ \ t)\ (1)$

$\therefore S_h = S_v$ Q.E.D.

Since the intensities of the vertical and horizontal shears are equal it is evident that the distribution of the vertical shear can be determined by the application of the formula for the intensity of the horizontal shear, viz:

$V = \dfrac{S\ Q}{b\ I}$, from which it at once follows that the assumption of uniform distribution of the vertical shear over the web is nearly correct since Q is the only vertical in the equation affected by the distance from the neutral axis, and its value changes much more slowly than does the distance from the axis. The numerical examples given in Article 64 show the degree of approximation of this assumption for certain typical girders.

In many girders the thickness of the web is determined by a reasonable restriction upon the minimum thickness imposed to prevent undue corrosion. For railroad bridges it is common to specify that the web shall not be less than 3/8" thick and this thickness often gives excessive strength. In consequence the necessity for a precise formula for the thickness of webs is less than for other portions of the girder.

62. PLATE GIRDER FLANGES. THEORY. The ordinary beam formula $M = \dfrac{f\ I}{y}$ applies to girders as well as beams. It is, however, an inconvenient form for use and is replaced in practice by

a less accurate but more easily applied formula. The formula recommended for flange design is as follows;-

$$A = \frac{M}{fh} - \frac{1}{12}\, t\, h, \qquad 62$$

In the application of this formula the tension flange is designed and the compression flange made like it.

The nomenclature is as follows;-

A = area in square inches of tension flange (through rivet holes.)
h = distance in inches between centres of gravity of flange.
f = allowable unit stress in bending.
t = thickness of web in inches.
h = depth of web in inches.
M = maximum bending moment on given section in inch pounds.

In the derivation of this formula the following additional nomenclature will be used.

I = total moment of gross cross section about neutral axis.
A_1= gross area of each flange.
h_2= depth out-to-out of flanges.
$I_{c.g.}$ = moment of inertia of each flange about its own centre of gravity.

The following equation for I may now be written.

$$I = 2I_{c.g.} + 2A_1 \left(\frac{h}{2}\right)^2 + \frac{1}{12}\, t\, h_1^3$$

The term, $2I_{cg}$ is small in comparison with the other terms, as will be shown later and may be omitted without serious error, this being on the safe side. In consequence the value of $\frac{I}{y}$ may be written thus

$$\frac{I}{y} = \frac{\frac{2A_1h^2}{4} + \frac{1}{12}\, t\, h_1^3}{\frac{h_2}{2}} = \frac{A_1h^2}{h_2} + \frac{1}{6}\,\frac{th_1^3}{h_2}$$

$$\text{But } \frac{I}{y} = \frac{M}{f} \therefore \frac{M}{f} = \frac{A_1h^2}{h_2} + \frac{1}{6}\,\frac{th_1^3}{h_2}$$

$$\text{hence } A_1 = \frac{M}{f}\,\frac{h_2}{h^2} - \frac{1}{6}\,\frac{th_1^3}{h_2}\,\frac{h_2}{h^2} = \frac{M}{f}\,\frac{h_2}{h^2} - \frac{1}{6}\,\frac{th_1^3}{h^2}$$

In ordinary girders the value of h is seldom larger than that of h_1 while it is usually considerably smaller. * If $\frac{h_1}{h}$ be assumed equal to unity the last term in above equation will therefore ordinarily be less than its true value.

$$= \frac{M}{fh} \left(\frac{h_2}{h}\right) - \frac{1}{6} th_1 \left(\frac{h_1}{h}\right)^2$$

This term in all ordinary girders is small compared with the term involving M, and the slight change in its value by the above approximation will affect the value of A but little, and that on the safe side since it reduces the value of the negative term. This approximation will therefore be made.

The assumption that $\frac{h_2}{h}$ = unity will also be made. This is on the unsafe side since h is always less than h_2 and to assume it equal gives a smaller value for A_1 than is needed. The error in making this assumption is largest in shallow girders having large flanges as may be seen in the numerical examples given later.

By making the above approximation the formula becomes

$$A_1 = \frac{M}{fh} - \frac{1}{6} th_1$$

For material in compression it is customary to make no deduction whatever for holes since it is assumed that the rivet which is driven while hot and ordinarily under high pressure fills the hole so completely as to become an integral portion of the material. This is open to some doubt, for material of considerable thickness or for hand-driven rivets, and may be vitiated at any section by a loose rivet, but for most cases this assumption is probably a reasonable one. For sections in tension full allowance for rivet holes must be made since under no circumstances can tension be transmitted through a rivet hole.

The last term in formula 61a represents the bending resistance of the gross web. In the webs of girders there are usually vertical rows of rivets for the connection of other girders and stiffener ∠s and as these may occur at the section carrying maximum moment they must be considered.

---

* Most specifications forbid the use in design of a value for h greater than h_1 even if it actually exists. It is good practice to proportion girders so that such a condition will not occur.

To allow for these rivet holes, it may be assumed that a vertical row of holes one inch in diameter and 2-3/4" apart may occur in the tension-half of the web. This would decrease the moment of inertia by 2/11 approximately, thus making the last term in the equation $\frac{9}{66} th_1$ or say $\frac{1}{8} th_1$.

Allowance for rivet holes in the tension flange must also be made. This may be done by substituting A for A_1, which is in reality equivalent to making allowance for rivet holes in both flanges. This may seem excessive but some excess is necessary since the section has been dealt with as solid and with neutral axis at centre when in reality the influence of holes in the tension portion is to move the neutral axis from the centre, thus diminishing the moment of inertia and increasing the distance from neutral axis to extreme fibre. The substitution of A for A_1 is, however, more than sufficient for this purpose and helps to diminish the error made in placing

$$\frac{h_2}{h} = \text{unity}.$$

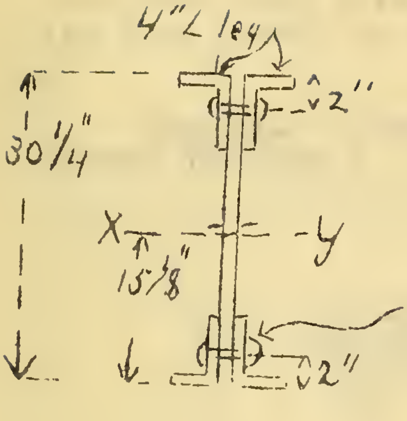
The above modification gives this formula

$$A = \frac{M}{fh} - \frac{1}{8} th_1$$

which is in the form adopted by many engineers. The last term in this equation represents the resistance of the web to bending. Owing to the difficulty in making a satisfactory web splice many engineers disregard this web resistance and use the formula $A = \frac{M}{fh}$. It is believed, however, that ample provision has been made for poor web splicing for long girders and for the greater approximation in short and shallow girders by changing this term to $\frac{1}{12} th$, the value given in formula 62a.

63. COMPARISON OF BENDING FORMULAS. In order to show the degree of approximation in using formula 62a in place of the more correct formula, $M = \frac{fI}{y}$ the problems which follow have been inserted.

Problem. Compute allowable bending moment, M, for the railroad bridge stringer shown in Fig. 63a. Assume no intermediate web stiffeners, and hence only one flange hole (flange rivet) in tension half of girder. Allowable unit stress = f.



Flange $\angle$s 6" x 4" x $\frac{1''}{2}$

Web 30" x $\frac{1''}{2}$

$h = 30.25'' - 3.98'' = 26.27''$

Allow for this rivet hole only. Diam. of hole, 1", for 7/8" rivet. Area to be deducted $= 1'' \times \left(\frac{1''}{2} + \frac{1''}{2} + \frac{1''}{2}\right) = 1\frac{1}{2}$ sq. inches

Fig. 63a

Net area of cross section				sq. in.
Top $\angle$s	2 @ 4.75		=	9.5
Bottom $\angle$s	2 @ (4.75-0.5)		=	8.5
Web	29" x $\frac{1''}{2}$		=	14.5
		Total		32.5

Centre of gravity of cross section above axis x y

$$= \frac{1'' \times 1\frac{1''}{2} \times 13\frac{1}{8}}{32.5} = .6'' =$$ influence of rivet hole on position of neutral axis.

M by exact method.

Ixy for webs $= \frac{1}{12} \cdot \frac{1}{2} \cdot 30 \cdot 30 \cdot 30$ $= 1125$

Ixy for $\angle$s $= I_{c.g}$ of $\angle$s $+ \frac{Ah^2}{2} = 4 \times 17.4 + 4 \times 4.75 \times (13.12)^2$ $= 3340$

Total I for gross section about axis xy $= 4465$

Deduct for rivet hole $1 \times 1\frac{1}{2} \times (13.12)^2$ $= 258$

4207

Correction for I about neutral axis $= 32.5 \times .6^2 =$ 12

I for net section about neutral axis $=$ 4195

$$\frac{I}{y} = \frac{4195}{15\frac{1}{8}'' + .6''} = 266$$

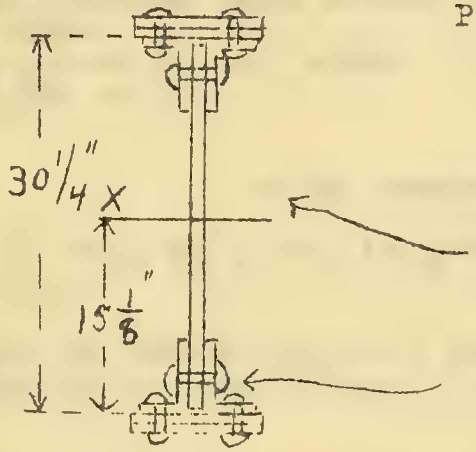
$$\therefore M = f\frac{I}{y} = 266\, f.$$

M by FORMULA 62. By transformation of terms in the formula M is found to be given by the expression $M = (A + \frac{1}{12} th1)\, fh$.

$$= (8.5 + 1.25)(f)\ (26.27) = 256.f$$

Since the allowable moment as computed by formula 62 is less than that given by the more exact method, the formula for this girder is on the safe side. The error is about $3\frac{1}{2}$%.

Problem. Compute allowable bending moment for the same girder assuming 2 - 10" x $\frac{1}{2}$" plates to be added to each flange.



Plates 10" x $\frac{1}{2}$"

<s 6" x 4" x $\frac{1}{2}$"

y web 30" x $\frac{1}{2}$"

Assume these 2 rivets only in a cross section.

Fig. 63b

Computation of h

Cross area of one flange = 19.5 sq.in.
From back of <s to c.g.of <s = 1.99"
(from hand-book)hence c.g.of flange
from c.g. of <s $= \frac{10x(1.99 + 0.5)}{19.5} = 1.27"$

$\therefore$ h = 30.25 - 2x(1.99" - 1.27")
= 28.81"

Net area of cross section

Top <s 2 @ 4.75	=	9.5
Bottom <s 2 @ (4.75-0.5)	=	8.5
Top places 2 @ 5.0	=	10.0
Bottom plates 2 @ 4.0	=	8.0
Web. 30 x 1/2	=	15.
Total		51.0

Allowance for rivet holes = 1" x $1\frac{1}{2}$ x 2 = 3 sq.inches
Hence Centre of gravity of cross section above axis xy.

$$= \frac{3. \times (15\frac{1}{8} + \frac{1}{4} - 1)}{51''} = 0.9''$$

M by exact method.

Ixy for web = 1125
Ixy for ∠s = 3340
Ixy for plates = $20 \times (15\ 5/8)^2$ = 4880

Total I for gross section about axis xy = 9345
Deduct for rivet holes $3 \times (15\ 3/8)^2$ = 709
Correction for I about neutral axis = 51×0.9^2 = 41
750

I for net section about neutral axis = 8595

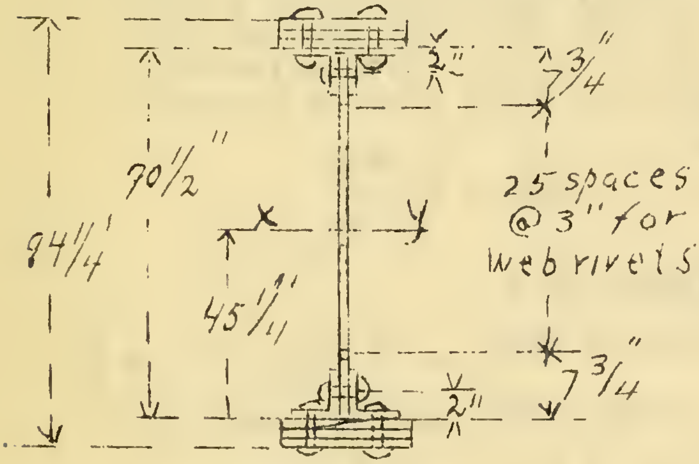
$$\frac{I}{y} = \frac{8595}{16\ 1/8 + 0.9} = 505 \quad \therefore \; M = 505\ f.$$

M by formula 62.

$$M = (A + \frac{1}{12} th_1)\ fh = (8.5 + 8.0 + 1.25)(f)(28.81) = 511.4f.$$

Here the formula errs on the unsafe side, the error in this case being slightly over 1 percent.

Problem. Compute allowable bending moment for girders shown in Fig. 63c. Assume vertical row of rivets, 3" centre to centre, at point of maximum moment.



Cover plates 16" x 5/8"
Flange ∠s 8" x 6" x 3/4"
Web 90" x $\frac{1}{2}$"

Assume that all three rivets in tension flange are in the same cross section

Fig. 63c.

Net area of cross section.

Top ∠s 2 x 8.44 = 16.88
Bottom ∠s 2 x (8.44 - 2 x 3/4) = 13.88
Top plates 3 x 10 = 30.00
Bottom plates 3 x (10-1.25) = 26.25
Web $(90-14)\frac{1}{2}$ = 38.00
Total = 125.01

Computation of h.

Back of ∠s to c.g. of ∠s = 1178″

$$\frac{\text{Moment of plates about c.g. of } \angle s}{\text{Cross area of flange}} = \frac{30 \times 2.71}{46.8} = \frac{1.73''}{.05}$$

$$\therefore \quad h = 90.5 - 0.10 = 90.40$$

Moment about xy of rivet holes in tension half of girder is as follows:

$$\text{Web. } \frac{1''}{2}(43.15 + 37.5 + 34.5 + \text{---} + 1.5) = \frac{1}{2} \times 297 = 148.5$$

$$\text{Flange } 1\ 1/2'' \times (43 \frac{1''}{4} + \frac{1''}{8}) + 3.75\ (46.18) = \underline{305.7}$$

Total 454.2

$$\text{Distance of neutral axis above } xy = \frac{454.2}{1250} = 3.63$$

Maximum value of $y = 50.75$

M by exact method

Ixy for web = 30375
Ixy " ∠s = 63794 + 112 = 63906
Ixy " plates = 127955 + 17 = 127972
I of gross section about axis xy = 222253
Deduct for rivet holes 4237 + 13856 = 18093
I of net section about axis xy = 204160
Correction for I about neutral axis = 1647
I of net section about neutral axis = 202513

$$\frac{I}{y} = \frac{202513}{50.75} = 3990 \quad \therefore M = 3990f$$

M by formula 62.

$$M = (A + \frac{1}{12} th_1)\ fh = (1388 + 26.25 + 3.75)\ 90.40f = 3967f.$$

In this case the error equals $\frac{1}{2}$ % on the safe side.

The examples that have been given show that formula 62 gives a very close approximation to the value obtained by the ordinary beam formula $M = \frac{fI}{y}$. It is much more convenient to use, since by it the required flange area can be directly computed, after the web is determined, by estimating the value h which can be done by the experienced commutor with little error. The actual application of the formula to the design of a girder will be shown later.

64. EXAMPLES OF SHEAR DISTRIBUTION. To show the degree of approximation involved in the ordinary assumption that the vertical shear is distributed uniformly over the net area of web, the maximum intensity of shear will be computed for the girders in the last article. This occurs at the neutral axis since the statical moment of the portion of a girder between the extreme fibre and that axis is larger than the statical moment about any other axis. It will be computed in each case for the portion of the girder above the axis/ although the same numerical result would be obtained by considering the portion below the axis since the statical moment of the entire section about any axis must equal zero.

Problem. Compute maximum intensity of shear for the girder shown in Fig. 63a.

In the computation already made for this girder, it was assured that no intermediate web stiffeners were needed and that in consequence there would be only flange rivets at the point of maximum moment. In the determination of the shear however a vertical row of rivets must be assured at the point of maximum shear since each stiffeners will surely be needed. In consequence the values of I, etc. previously found must be modified somewhat.

Let a vertical row of web rivets be assumed with 3" spacing beginning 7 5/8" from back of flange angles.

There will then be three rivets below the neutral axis in addition to the flange rivet already considered.

The net area of the cross section will cross equal 32.5-1.5 = 31.0 sq. in.

Its centre of gravity above axis xy will be given by the expression.

$$\frac{1\frac{1}{2} \times 13\frac{1}{8} + \frac{1}{2}\left(7\frac{1}{2} + 4\frac{1}{2} + 1\frac{1}{2}\right)}{31.0} = \frac{26.43''}{31} = 0.85''$$

I_{xy} for gross area of web and $\angle$s $= 4465$

Deduct for rivet hole $258 + \frac{1}{2}\left(7\frac{1}{2}^2 + 4\frac{1}{2}^2 + 1\frac{1}{2}^2\right)$ $\underline{297}$

I for net section about xy 4168

Correction for I about neutral axis $= 31 \times 0.85^2$ $= \underline{22}$

I for net section about neutral axis $= 4146$

Q for $\angle$s $= 9.5 \times (15\frac{1}{8} - 2.85)$ $= 116.6$

" " web $=(15 - 0.85)(\frac{1}{2})\left(\frac{15 - 0.85}{2}\right)$ $= \underline{50.0}$

166.6

$$U = S \frac{166.6}{4146 \times \frac{1}{2}} = .0804\ S$$

From the assumption that the shear is uniformly distributed over 3/4 the gross area of the web the following result is obtained.

$$u = \frac{S}{3/4 \times 15} = .0888S$$

The value thus obtained is on the safe side by about 10%.

For the girder shown in Fig 63 b the same results would be obtained since the cover plates should not be carried to the end where the maximum shear occurs.

The following results are obtained for the girder shown in Fig. 63c, the cover plates being ignored for the reason previously given.

Net area 68.76 sq.in.

Centre of gravity above xy $= \dfrac{148.5 + 132.18}{68.76} = 4.08''$

I x y for gross area of web and ∠s $= 94281$
Deduct for rivet $4237 + 5826$ $\quad 10063$
I of net section about xy $\quad 84218$
Correction for I about neutral axis
$= 68.76 \times 4.08^2 = 1140$
I of net section about neutral axis $= 83078$

Q of ∠s $= 16.88 \times 39.39 = 665$
" of web $= (45-4.08)\left(\frac{1}{2}\right)\dfrac{(45-4.08)}{2} = 419$
$\quad 1084$

$$\therefore V = \frac{1084 \quad s}{83078 \times \frac{1}{2}} = .0261\ S$$

By assumption that shear is distributed over 3/4 gross area of web, the following value would be obtained.

$$V = \frac{S}{3375} = .0296\ S$$

This result is again on the safe side by about 10%.

If in either of the cases considered the shear had been assumed as distributed uniformly over the gross area, a considerable error on the unsafe side would result.

Although the two girders considered do not represent extreme cases, it is believed that the results are represented and that in all cases the assumption that the shear is distributed uniformly over 3/4 the gross area of the web is a safe and reasonable working hypothesis.

It should be noted that some engineers consider the shear as distributed uniformly over the gross area of the web, but this is inconsistent with the results obtained for these typical girders and should not be adopted unless the shearing unit is modified. The fact that the rivet may perhaps carry shear as well as compression should not be regarded as a reason for assuming the shear uniformly distributed over the gross area since it is the influence of the flange on the distribution of the shear which is really the important factor.

For I beams the shear may be treated in the same manner. The values obtained for a 10" I-25 lbs. per ft. are as follows:

$$\text{By formula 53a} \quad v = \frac{14.02}{0.31 \times 122.1} S = .370 S$$

$$\text{By common assumption} \quad v = \frac{S}{2.325} = .430 S$$

The error in this case is on the safe side but had the gross web area been used the error would have been on the unsafe side.

For a 24" I - 80 lbs. per ft. the corresponding values would be .097S and .111S the error in this case being also on the safe side.

As it is seldom that the shear in I beams is an important factor in the design this error is of not of great importance.

65 ALLOWANCE for RIVET HOLES. In the design of girders it is necessary to make in advance of the detailed drawings due allowance for the tension flange rivet holes. No accurate rule for doing this can be given since the actual reduction of strength by rivet holes needs more thorough experimental investigation than it has yet received. The following rules may, however, be used as a guide.

1st. For flanges with cover plates and angles with legs 4" or over, in width assume that the rivet holes may occur in the same section, these holes being as shown in Fig. 65a.

2nd. For flanges with flange angles of 4" or less in width, and with cover plates deduct two holes from each section as shown in Fig. 65b.

3rd. For flanges without cover plates deduct one hole from each section as shown in Fig. 65c.

In all cases the rivet hole should be assumed as having a diameter 1/8" more than the nominal diameter of the rivet. This is necessary since the hole is usually punched with a diameter 1/16" greater than that of the cold rivet, and the edges of the hole may be damaged somewhat in punching. The rivet commonly used in structural work is 7/8" diameter, hence for these rivets the hole should be assumed as one inch in diameter. In light work 3/4" or 5/8" rivets are occasionally used and in very heavy girders one inch units are sometimes employed.

It should be stated that while it is seldom that more than three rivet holes actually occur the same section of the tension flange, the fact that a zig-zag section through holes not in the same cross section may have a less net area than any given cross section must be considered. For example the section AB in Fig. 65d. may have a much smaller net section than the cross section AC and if the distance AB is small may have a net area but little if any less than that on a section like DE. If desirable the actual net area or a section like AB may be computed or determined graphically, although experimental results are lacking to show that the strength of the flange varies directly with such an area. It is, however, wise to make liberal allowance for rivet holes and if the pitch CB is less than 2 1/2" to ordinarily allow for three holes in the section as shown in Fig. 65a.

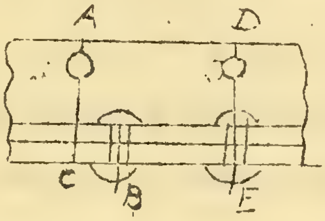


Fig. 65d.

Attention may also be called to the fact that while the maximum moment on a girder ordinarily occurs at only one section and that at this section the rivet pitch may be a maximum, a maximum flange stress is developed wherever a cover plate ends and the rivet pitch at such a point may be little, if any, larger than the minimum allowable pitch.

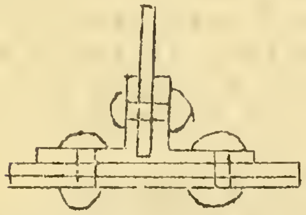


Fig. 65a.
3 holes in section.
= 2 in each $\angle$ and 2
in each plate.

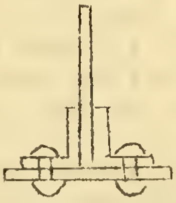


Fig. 65b
2 holes in section
= 1 in each $\angle$ and
2 in each plate.

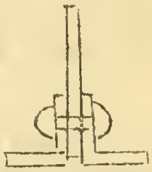


Fig. 65c
1 hole in section.
= 1 in each $\angle$

66 EXAMPLE OF GIRDER DESIGN. An example of the complete design of the cross section of a girder will now be given.

Problem. Determine cross section of a girder to carry a maximum bending moment of 1,250,000 ft. lbs. and a maximum shear of 160000 lbs; Depth of girder back to back of flange $\angle$s is to be 48 1/2".

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Allowable unit stress Bending 16000 lbs.per.sq.inch
Shear 12000 " " " "

Web. net area of cross section $= \frac{160000}{12000} = 13.33$ sq. inches.

Depth of web 48". Thickness of web $\frac{13.33}{3/4 \times 48} = 0.37''$ or 3/8"

Flange Assume h to equal depth of web = 48"

$$\text{Trial } A = \frac{1{,}250{,}000 \times 12}{16000 \times 48} - \frac{1}{12} \cdot 3/8 \cdot 48 = 19.53 - 1.50 = 18.03 \, \square''$$

Try 2 $\angle$s 6" x 4" x 5/8" @ 5.86 = 11.72 − 2.50 = 9.22 sq.in.
2 plates 14x3/8 @ 5.25 = 10.50 − 1.50 = 9.00 " "

(Rivets in flange assumed as in Fig. 64a 18.22 " "

Compute h for this section.

Centre of gravity of $\angle$s from back of $\angle$s = 1.03 (from handbook)

Correct to allow for cover plates by $\frac{10.5 \times 1.40}{10.5 + 11.72} = 0.66''$

$\therefore h = 48.5'' - 2 \times (1.03 - 0.66) = 47.8''$

Hence original assumption for value of h, while slightly too large is sufficiently accurate and section may be used as computed.

67 RIVETS and RIVETED JOINTS. The flange rivets form the only connection between the flange and the web, hence the determination of the proper size and distance apart of these rivets is an essential feature in girder design. The diameter of the rivet is ordinarily fixed by practical considerations the common practice having been stated in Article 64. The distance apart of the rivets has to be computed.

Thorough treatment of rivet spacing is found in treatises upon mechanics and will not be given here. The essential points with which the bridge designer must be thoroughly conversant are as follows;–

A riveted connection may fail in one of the following ways;

a. By the shearing of the rivets.

b. By the crushing of the rivets or of one of the pieces upon which they bear.

Under a it should be noted that the allowable shearing value of the rivet may be found by multiplying its cross-section area by the allowable shearing stress per square inch, and that the area of a 7/8" rivet is 0.60 sq. inches, and of a 3/4" rivet 0.44 sq. inches.

In designing rivets to resist shear the plane upon which the maximum shear occurs must always be determined. If the maximum shear is equally distributed over two planes the rivet is said to be in double shear.

The permissible bearing, or crushing, strength of a rivet against a given plate is determined by multiplying the allowable bearing strength per square inch by the diameter of the rivet and the thickness of the plate in question.

c. By the tearing of the rivets through one of the connected pieces.

To prevent failure mentioned under c use the following empirical rule;- Rivets may not be spaced closer than three times the diameter, and the distance of a rivet from the edge or end of a piece may not be less than 1 1/4" for a 7/8" rivet if the edge in question be rolled or planed, or 1 1/2" if it be sheared though where possible this distance should be at least twice the diameter of the rivet. For other sizes of rivets proportional allowances should be made.

The two following examples show the application of these rules to some simple cases.

Problem;- Determine number of 7/8" rivets needed in row a to connect plates shown.

Allowable shearing stress = 7500 lbs. per. sq. inch
" bearing " = 15000 " " " "

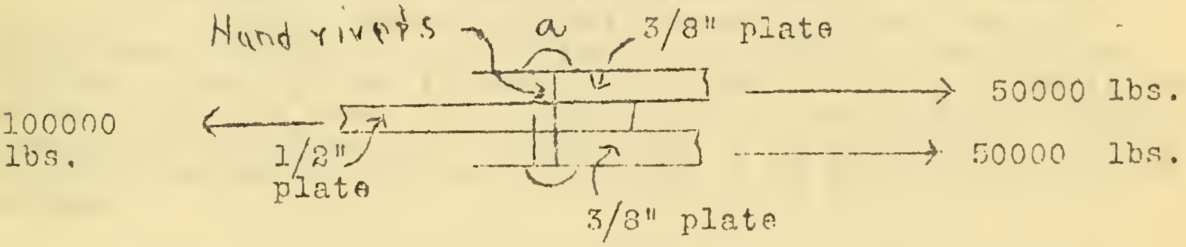


Fig. 67a.

Solution;-

The maximum shear on a plane through the rivets = 50000 lbs., As the rivets are 7/8" diameter, one rivet will stand in shear 7500 x 0.6 = 4500 lbs.; hence, if the strength of the joint is limited by the shearing strength of the rivets there are needed $\frac{50000}{45}$ = 11 (+) or 12 rivets. These rivets are limited in crushing strength by the 1/2" plate which carries 100000 lbs. The value of the rivet in bearing against this plate equals

7/8 x 1/2 x 15000 = 6560 lbs. and the number required = $\frac{100000}{6560}$ = 15 (+) or 16. As this is larger than the number needed to prevent shearing 16 rivets must be used.

Problem. Determine number of 7/8" rivets required in this joint to connect plates shown. Use same rivet values as previous problem.

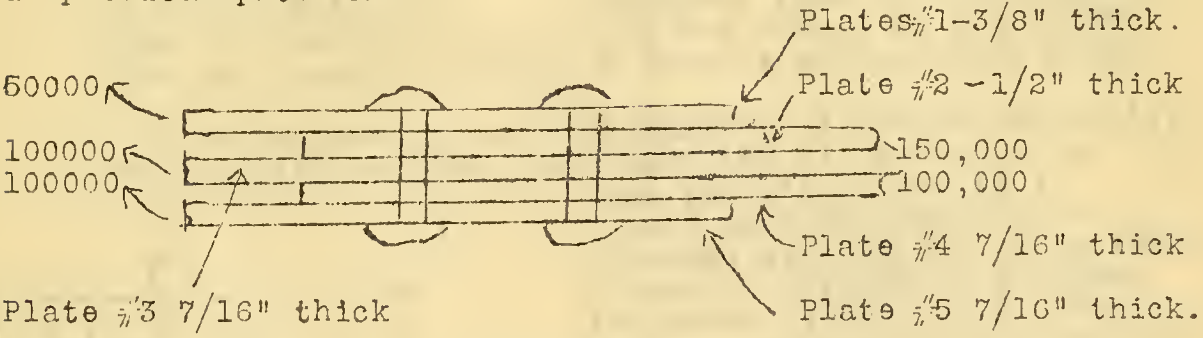


Fig. 67b.

Maximum shear as plane through rivets is 100000 lbs and occurs between plates 2 and 3, and 5 and 4. Number of rivets needed to carry this shear = $\frac{100000}{4500}$ - 22 (+) or say 23.

The bearing strength is evidently limited by plate #2 which carries 150,000 lbs and has a thickness of 1/2", this producing a greater stress on the rivets than would be the case for the 7/16" plate carrying 100,000 lbs.

The number required for bearing = $\frac{150000}{6560}$ = 15 + or 16, hence for this joint the number of rivets is limited by shear.

The examples just given illustrate methods of computing rivets serving to connect plates carrying direct stress. Sometimes, however, it is necessary to transmit tension as well as direct stress by means of rivets. Such a condition often occurs in steel plane building construction where the connection of girder and columns has to be given considerable rigidity to permit the wind stresses to be properly transmitted.

The condition arising in such a case is represented diagrammatically by Fig. 67c in which the load P is applied at a distance a from the centre of the group rivets thus producing a tension Pa which must be carried by the rivets in addition to the direct load P.

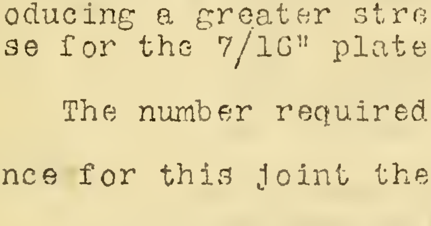


Fig. 67c.

The ordinary method of computation in such a case is to assume that the direct load is distributed uniformly amongst the rivets, and that the stress in the rivet to tension varies directly with its distance from the centre of gravity of the groups of rivets and acts at right angles to the line joining the centre of gravity with the rivet under consideration. As the resistance to torsion equals the stress

in the rivet multiplied by its distance from the axis of rotation it is apparent that by this assumption the resistance varies as the square of the rivets' distance from the centre of gravity of the rivet group.

If the torsion were produced by a couple as in Fig. 67d then the vertical load upon the rivets would be zero and the rivets at a should each be computed as stressed equally and up to the allowable working load while the other rivets would carry such proportion of the working stress as the distance CB to CA when C is the centre of gravity of the group of rivets. The resistance of such a group of rivets to a couple may therefore be expressed materially as follows:-

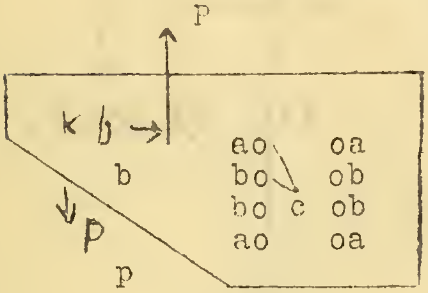


Fig. 67d

Let r = the allowable working value of the most stressed rivet.

" I = sum of square of the distance from the centre of a rivet to the centre of gravity of the group of rivets.

" d = distance from centre of gravity of group of rivets to the most stressed rivet.

" R = resistance of the group of rivets.

$$\text{Then } R = \frac{r}{d} \quad I$$

For the case shown in Fig. 67c the above method must be modified to allow for the effect of the vertical load on the rivets. To make this correction it is only necessary to determine the allowable resistance to torsion consistent with the rivet carrying its share of the vertical load.

This may best be done graphically by the method indicated in Fig. 67d in which the rivets at a are found to be the limiting cases since these rivets can offer only 11700 lbs resistance to torsion for a a total working value of 15000 lbs while the other rivets could all offer a much greater resistance than this if called upon. Hence for this case 11700 lbs. should be substituted for r and the resistance of the joint to

$$\text{bending} = \frac{5850}{\sqrt{3^2 + 4\ 1/2^2}} \quad 4\left[\left(3^2+4\ 1/2^2\right) + \left(3^2+1\ 1/2^2\right)\right] =$$

175000 in. lbs.

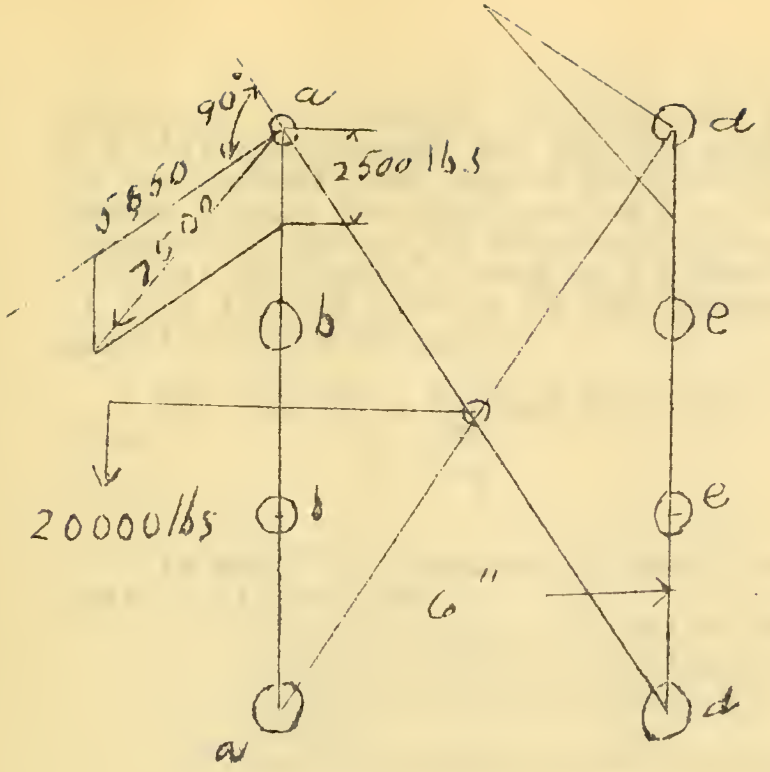


Fig. 67d.

In this figure the vertical load is 20000 lbs. hence each rivet is assumed to carry 2500 lbs. of this. With an allowable working stress on the rivet of 7500 lbs the component at a perpendicular to the line a c is 5850 lbs. The rivets at d, b and c would be found to have larger composition, hence the rivets at a are the highest stressed and consequently give the limiting value, v, in the equation. Tables giving the resistance to torsion of various groups of rivets have been prepared by E.A. Rexford and are published by Engineering News Publishing Co.

68 FLANGE RIVETS. ORDINARY METHOD OF COMPUTATION. Since it is through the rivets that stress is transmitted into the flanges it is evident that the number of rivets needed is a direct function of the flange stress which in turn is a direct function of the moment, hence the distance apart of the rivets or the pitch (the pitch is to be considered the distance apart of the rivets measured along the flange, i. e., in the direction of the stress) varies as the moment. In a girder supported at the ends and carrying a uniform load the curve of moments is a vertical parabola, with its vertex on the vertical line passed through the centre of the span, hence the variation in the moment is a minimum at the centre and a maximum at the ends and the rivet pitch follows the same law. This is also approximately true for such girders when loaded with other than uniform loads.

A knowledge of the variation of the pitch is, however, insufficient; it is necessary to determine the pitch itself. The following method of doing this is obvious;- Compute the total stress in the flange at any section and compute also the total stress at a section one inch from the first section; the difference between the two stresses gives the increase in flange stress per longitudinal inch at that portion of the girder and this increase must be carried into the flanges through the rivets. If one rivet can carry p lbs. and if the increase in flange stress is x lbs, the proper rivet pitch to use in that portion of the girder is $\frac{p}{x}$.

If the portion of the bending moment carried by the web be neglected in the determination of rivet pitch, this error being small and on the safe side, the increase in flange [illegible] figure anal[illegible] [illegible] is [illegible] the [illegible] [illegible] [illegible] [illegible] [illegible] the [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] the [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] the [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] the [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] the [illegible]

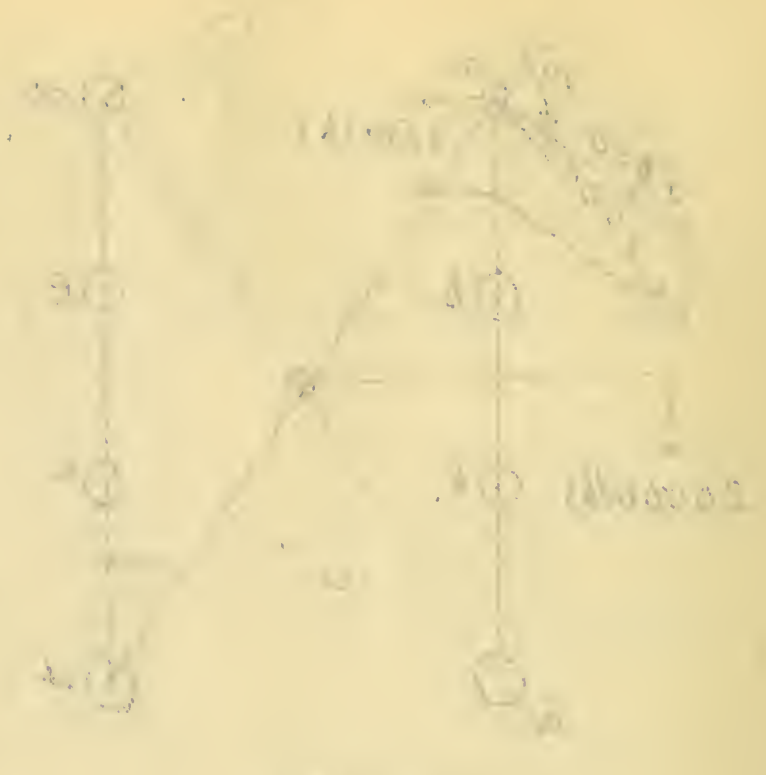


Fig. 185.

68 placed either [illegible] [illegible] [illegible] [illegible] It is through the [illegible] [illegible] [illegible] [illegible] [illegible] the [illegible] It is evident that the [illegible] [illegible] [illegible] [illegible] [illegible] of the figure [illegible] [illegible] [illegible] [illegible] [illegible] of the [illegible] [illegible] [illegible] [illegible] [illegible] of the [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] of the [illegible] [illegible] the [illegible] [illegible] [illegible] of the [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] the [illegible] [illegible] [illegible] of the [illegible] [illegible] with the [illegible] of the [illegible] [illegible] through the [illegible] of the [illegible] [illegible] [illegible] [illegible] [illegible] of the [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] than [illegible] [illegible]

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If the position of the bending moment referred to the [illegible] [illegible] be neglected in the determination of fixed points. This [illegible] being small and [illegible] the [illegible] the [illegible] in [illegible] stress may be found by dividing the increase in moment by the distance between the centres of gravity of the flanges. It has already been stated that the first derivative of the moment equals the shear and as the first derivative equals the increase in moment it therefore follows that the rate of increase in flange stress at a given section equals the shear at that section divided by the distance between centres of gravity of the flanges.

The following formula for rivet pitch may therefore be given;-

$$p = \frac{Rh}{S} \qquad 68$$

In which p = maximum allowable pitch in inches at section under consideration.

R = allowable load on rivet in pounds (usually the value of rivet in bearing on web).

h = distance between centres of gravity of flanges in inches.

S = maximum external shear on given section in pounds.

Owing to the fact that if the distance between rivets be too great the pieces may tend to fail in detail, especially in a compression piece, it is customary to specify a maximum pitch not greater than 6" or 16 times the thickness of the thinnest plate connected. This restriction is frequently the determining factor in settling the rivet pitch.

This method is applicable only to rivets through the vertical leg. These are the rivets through which all the stresses enters the flange. The rivets through the horizontal leg serve to transmit a part of this flange stress into the cover plates and in consequence may have a larger pitch. It is customary, however, to use the same pitch for the vertical as for the horizontal rivets, hence the method given is, in general, all that is necessary.

One special case must, however, be mentioned as it is of frequent occurrance, viz., where the whole load or a part of it is transmitted into the girder directly through the flanges such being the case in railroad bridge stringers, in girders carrying brick walls, etc. The results of such a loading is to impose a vertical load upon the rivets as well as a horizontal thrust, the stress per rivet being thereby increased and the allowable rivet pitch decreased. In solving such a case it is necessary to find the resultant of the increase per inch in the horizontal flange stress and in the vertical load per longitudinal inch, and to divide the value of the rivet by this resultant; the quotient thus obtained equals the pitch.

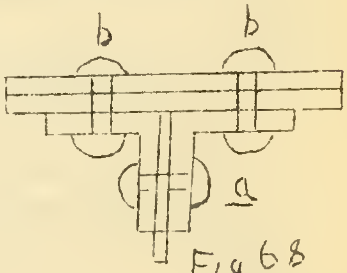


Fig 68

69. FLANGE RIVETS. PRECISE METHOD OF COMPUTATION. The method represented by equation 68 is the approximate method of figuring rivet pitch which is almost universally used in plate girder design. In order to thoroughly understand the question of rivet pitch and to be able to properly figure the pitch in other cases which may arise such as rivet pitch in columns carrying bending, etc., it is necessary to develop a more exact method. Such a method may also be well employed in investigating existing girders the strength of which may be in doubt. To obtain such a method the formula for horizontal shear may be used.

Referring to Fig. 68 it is evident that the function of the rivets at a is to prevent the flange angles from sliding along the web; that is, the rivets resist the horizontal shearing force of the angles. Hence, if this force can be computed the rivet pitch necessary to withstand it can be determined. This computation can be easily made by multiplying the intensity of the longitudinal shear by the thickness of the web, using for Q in the determination of the intensity the statical moment about the neutral axis of the angles and cover plates combined but but not of the portion of the web included between the flange angles since the stress in this is carried by the web itself. This gives the increase in shearing force per longitudinal inch in the flange which equals the increase in flange stress and this can be used as before in figuring the pitch. By the same method the correct pitch for the rivets at b can be determined by computing Q for the cover plates only.

70 RIVET PITCH COMPUTATIONS. EXAMPLES. To illustrate the application of these methods the girder shown in Fig. 63c will be considered.

Problem. Determine the rivet pitch required at a section where the shear is 300000 lbs.

The rivets through the vertical legs of the angles, will first be considered, assuming that the load as applied directly through the web and not on the flange. Let the bearing value per sq. inch of the rivets be taken as 24,000 lbs. and the shearing value as 12000 lbs. The strength of the rivet will then be limited either by bearing on the 1/2" web which equals 1/2 x 7/8 x 24000 = 10500 lbs., or by double shear which equals .60 x 12000 x 2 = 14400 lbs. As the bearing value is smaller it must be used.

Increase in flange stress per linear unit;-

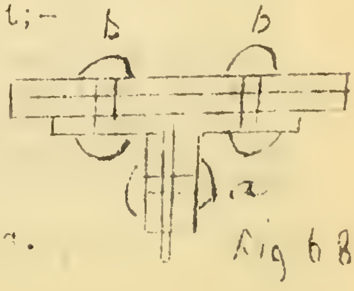


Fig 68

$$\text{Approximate method } \frac{300000}{90.4} = 3330.$$

$$\text{Exact method } 300000 \times \frac{1948}{202513} = 2880 \text{ lbs.}$$

Note that Q = 673 + 1276 for L's and cover plates and that I = 202513. See Article 63.

Since one rivet can carry 10500 lbs. the required pitch by the approximate method is

$$\frac{10500}{3320} = 3.18'' \text{ or say } 3''$$

and by the exact method $\frac{10500}{2880} = 3.65$ or say 3 1/2".

It is evident that the approximate method is decidedly on the safe side in this case.

Were the pitch found to be less than three diameters it would be necessary to locate the rivets in two rows as shown in Fig. 70a where a pitch of 2" is assumed.

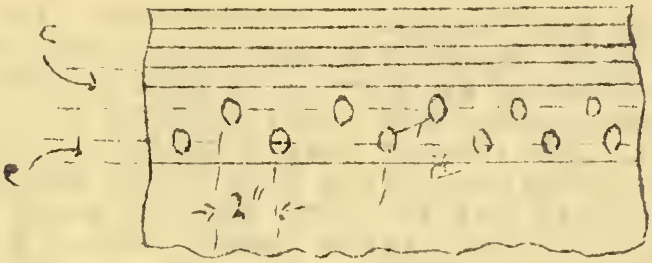


Fig. 70a.

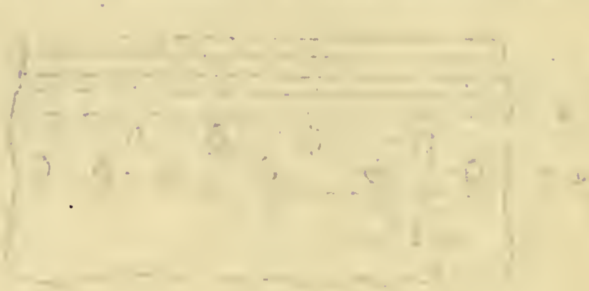
It will be noted that the pitch is taken as the distance between rivets measured along the axis of the angle. The distance apart of the rows of rivets must be sufficient to make the distance d equal to or greater than three times the diameter of the rivet or 2 5/8" for a 7/8" rivet. The distance e should not be less than 1 1/4" as already noted, and should preferably be 1 1/2" or 1 3/4". The distance c is determined by the amount of room needed for driving the rivet. The standard values for different angles is given in the steelmakers' hand-books.

For correct spacing on the vertical rivets the exact method must be used. The increase in stress per inch is

$$\frac{1276}{202513} \times 300000 = 1890 \text{ lbs.}$$

The value of each rivet in this case is evidently its strength in single shear, but as there would be two rivets in a cross-section the pitch may be obtained by dividing the value of one rivet in double shear by the increase per inch the pitch thus being found to be $\frac{14400}{1890} = 7.6''$ or say 7 1/2".

As this exceeds 6" (See article 68) the pitch for these rivets should be made 6" or less.



This example shows that the pitch of the vertical rivets may be considerably greater than that of the horizontal rivets. It is, however, usually made the same for practical considerations.

There remains one other case to be treated; that of a girder supporting a load on the upper flange. To illustrate the method required for this case let the girder shown in Fig. 63a be considered and let it be assumed that it is a railroad bridge stringer with ties resting directly upon the top flange. Let it also be assumed that the maximum wheel load crossing the stringer is 24000 lbs., that the maximum end shear including impact is 100000 lbs., and that the rivet pitch is to be figured at the end of the stringer. With the allowable unit stresses previously used the limiting value of the rivet is, 7/8 x 1/2 x 24000 = 10500 lbs. Using the approximate method the increase in flange stress is found to be

$$\frac{100000}{26.27} = 3800 \text{ lbs. per linear inch.}$$

This value must be combined with the vertical load, carried by the rivets. Since the rail has considerable strength as a beam it is evident that a wheel load will never be carried entirely by one tie, but will be distributed. The common assumption is that one wheel is distributed over three ties. If this assumption be made one wheel load will be distributed over the rivets in the space m. Fig. 70b. If the ties are 8" wide and spaced 6" apart in the clear, three ties will occupy a total distance of 36", hence the vertical load per inch which the rivets must bear is $\frac{24000}{36}$ or 660 lbs. neglecting the dead weight which is so small compared with the live load as to be negligible. To allow for impact this value should be doubled since these rivets are more directly affected by the shock of the locomotive than any other portion of the structure. To obtain the rivet pitch it is, therefore, necessary to divide the value of one rivet by the resultant of 3800 and 1332. The resultant may be obtained quickly and with sufficient accuracy by the graphical method indicated in the Fig. The resultant is found to be 4030, hence the proper pitch at the end is $\frac{10500}{4030}$ = 2.65 = 2 5/8" or say 2 1/2".

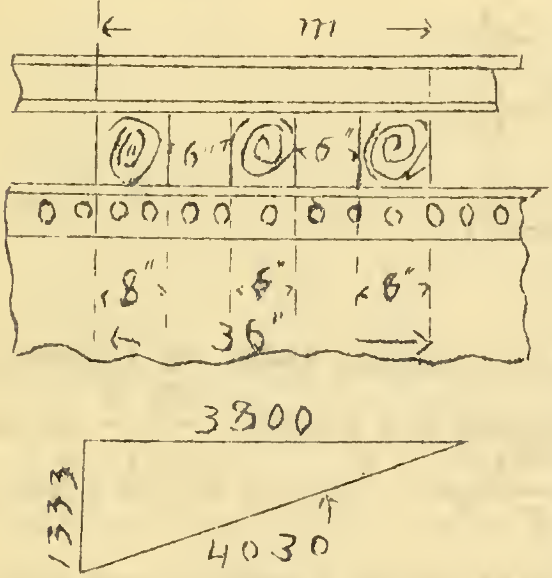


Fig. 70b.

This spacing can be used without difficulty since two rows of rivets should always be used in a 6" < leg, hence the actual distance apart of the rivets will be considerably greater than the nominal pitch. Were the 4" leg vertical instead of 3" leg the pitch of 2 5/8" could be used as this is the minimum allowable value, but the adoption of this minimum value except where unavoidable is, however, not recommended, a better

plan being to increase the depth or web thickness to permit larger pitch, or else to use a wider legged angle with two rows of rivets. It frequently happens that the determination of the flange section of girders is materially influenced by the question of rivet pitch, and the experienced designer will always look into this before selecting flange angles.

71 DIRECT WEB STRESSES. It has previously been shown that the intensity of the horizontal shear at any point in the web of a girder equals the intensity of the vertical shear and that these reach their maximum values at the neutral axis. Consider again an infinitesimal prism at the neutral axis. The shearing forces acting on this prism will develop internal forces of tension and compression, .The value of which may be found as follows;-

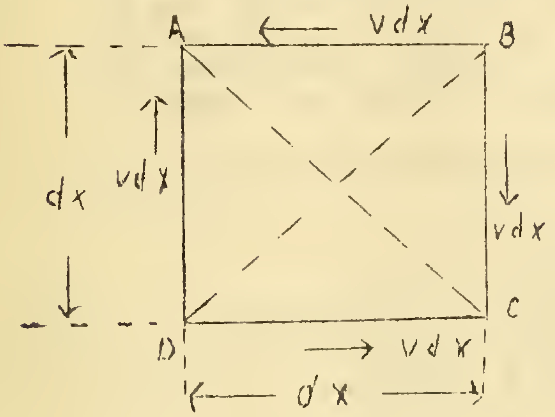
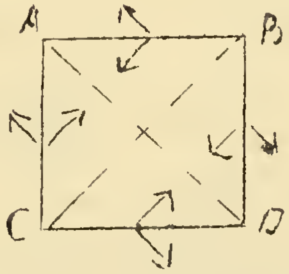


Fig 71a

Let the thickness of the prism at right angles to the paper be unity, and let v equal the intensity of the shear. Then the total shearing force on each side = v d x. Resolving these forces into components the value of each is found to be

$$\frac{vdx}{\sqrt{2}}$$ acting as indicated in Fig. 71b.



Fib Fig. 71b.

The effect of these components is to produce on D B a total tension $\frac{2\ vdx}{\sqrt{2}}$ Since the length of D B is $\sqrt{2}$ dx the intensity of the tension on D B $= \frac{2\ vdx}{\sqrt{2}\sqrt{2}\ dx} = v$.

In the same manner a compressive force may be shown to act on A C the intensity of which is also v. It therefore follows that at the neutral axis there exists a tension and compression acting upon planes at right angles to each other and at 45° to that axis and that the intensity of these forces is equal to that of the shear. If the prism in question has been taken above or below the neutral axis the above conditions would be modified somewhat through the introduction of direct fibre stresses which would then have to be considered. The effect of the shear in producing direct stresses would not be changed, that is the shearing forces would develop direct stresses as

before but the final value of the ~~tension~~ or compression upon any plane would have to be obtained by combining the direct stresses due to shear and the direct fibre stresses to bending.

The expression for the maximum direct stress for this more general case is developed in books on mechanics and is as follows:

$$p' = \frac{px + py}{2} + \frac{1}{2}\sqrt{4v^2 + (px - py)^2}$$

In this equation p' = intensity of the maximum direct stress occuring on any plane passing through a given point, px and py are the intensities of the direct stresses occurring at the same time on two rectangular planes passing through the same point, and v is the intensity of the shear on each of these two latter planes.

Fig. 71c illustrates this condition. It is evident that if point c is at the neutral axis of a beam subjected to bending but not to direct stress, px and py are both zero and p' = v.

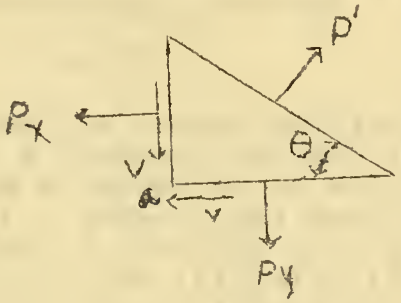


Fig. 71c.

The expression for the angle θ between the x plane and the plane upon which the maximum intensity occurs is as follows:—

Tan $\theta = \frac{v}{p}$, and therefore the case of the girder reduces to unity showing that $\theta = 45''^\circ$.

That tension and compression act as shown in the prism is also evident the distortion produced by the shearing forces. It is plain that under the action of the horizontal and vertical shear the prism which is rectangular when unstressed will take the shape shown in Fig. 71d.

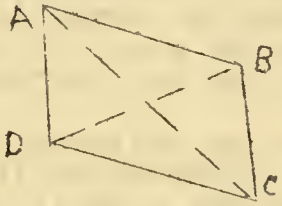


Fig. 71d.

and hence the line A C will be lengthened, and the line B D shortened. These changes can be produced only by tension and compression at 45° to the axis.

In plate girders the existence of compression at 45° to the axis is of considerable importance since if it be not recognized and proper means taken to provide for it failure will occur through sidewise buckling of the web. In reinforced

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concrete beams on the other hand the corresponding tension is the important factor since concrete is very weak in tension and means must generally be taken to provide against failure by rupture at 45° to the axis, either by steel rods placed at 45° to the axis, or by vertical stirrups.

The combination of the direct stress due to shear with that due to bending results in making the resultant compression in the web of a girder act in the direction indicated by the dotted lines in Fig. 71e. The shape of this line is, of course, dependent upon the relative value of the shear and the direct stress. At the end of the girder where the shear is a maximum and the bending moment a minimum, the line would lie at 45° to the axis throughout its entire length. At the section where the moment is a maximum the shear is zero hence here there is no direct stress in the web at the neutral axis, and above or below this axis the direct stress is parallel to it.

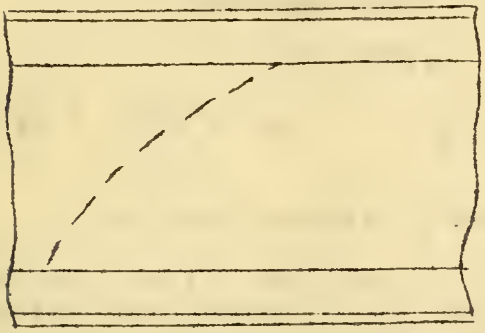


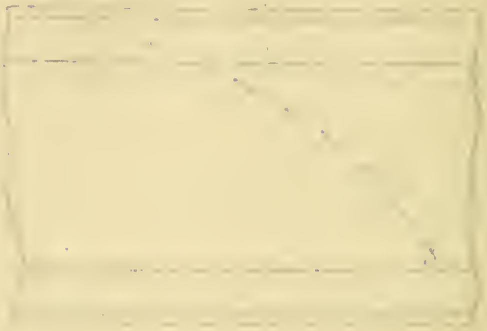
Fig. 71e.

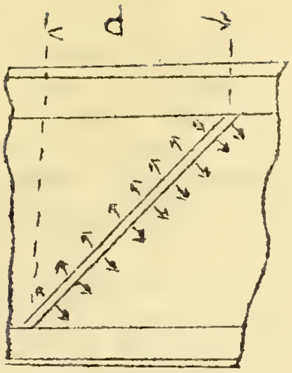
From the preceding discussion it is evident that in the design of a plate girder web attention must be given to the compression stresses which exist, and that a web designed to resist the shear alone may fail through buckling under compression. To prevent such action care must be taken to make the web of such thickness that there will be no danger of excessive compression or else the buckling tendency must be restrained by other means. The latter is the common method. and is accomplished by the use of stiffeners which are vertical angles riveted to the web and extending from top to bottom. Sometimes, however, it is more economical of material as well as of labor, to increase the web thickness rather than to use stiffeners.

72 WEB STIFFENERS. The subject of stiffener spacing is complicated and no accurate theory has yet been developed. Experimental results have been inconclusive but indicate that the ordinary methods of practice are safe if not precise. The only theoretical method that seems rational is to treat a strip of the web as a column and to make an assumption as to the influence of the remainder of the web upon this column.

By this method an equation can be deduced for the distance apart of the stiffeners which should be in a reasonable form and which if it does not give results exceeding the limits of good practice may be used with security.

Let Fig. 72a represent a portion of the web near the end of a girder where the shear is a maximum. Since the bending moment at the ends of the girder is small the direct web stresses act at approximately 45° throughout the entire depth of the girder, hence the strip of web to be considered is taken at a 45° slope. Its length is restricted by the flange angles and it is partially restrained against sidewise buckling by direct web tension at right angles to its axis as indicated in the figure by arrows;-





Let unity = width of strip.
" t = thickness of web.
" i = length of column.
" d = distance apart of stiffeners in clear.
" r = radius of gyration.

Fig. 72a.

Then $i = d\sqrt{2}$ and $r = \sqrt{\frac{I}{A}} = \sqrt{\frac{1}{12}\frac{t^3}{t}} = \frac{t}{\sqrt{12}}$

A column formula of the form $\frac{P}{A} = 16000 - c\frac{i}{r}$ will now be applied to the strip. Since c for the ordinary unsupported pin ended column may be safely taken as 70 it would not seem unreasonable to reduce this materially here since the column is fixed at the ends by the flange angles and held sidewise throughout its entire length by the direct web tension. The amount which it should be reduced to allow for these restraining influences is unknown but the value, c = 25, will be adopted as a conservative value giving results which do not exceed the limits of ordinary practice. Substituting this value for the constant and expression the value of $\frac{i}{r}$ in terms of t and d gives the following equation;-

$$\frac{P}{A} = 16000 - 25\frac{d}{t}\sqrt{24}$$

$$= 16000 - 120\frac{d}{t} \text{ (very nearly) in which}$$

$\frac{P}{A}$ equals the allowable intensity of axial compression in the columns. This for the case in question, equals the shearing intensity v, per square inch, hence the formula may be written thus

$$v = 16000 - 120\frac{d}{t} \qquad 72$$

This formula may be used in either of the following ways:

1°. To compute the allowable shear per sq. inch on the net section of the web for a given thickness of web and distance apart of stiffeners.

2°. To determine the required web thickness for a given shear and distance apart of stiffeners.

3°. To determine distance apart of stiffeners for given shear and web thickness.

Since the column lies at 45° to the axis no stiffeners will be needed if the value of d is found to be greater than the clear distance between flange angles. It is, however, customary to use stiffeners on all except shallow girders in order to stiffen the girder during fabrication and transportation and a common requirement is that the maximum clear distance between stiffeners shall be the depth of web plate between flange angles, and shall not be greater than 5 feet.

For that portion of the girder where the bending moment is large and the shear relatively small the web conditions differ materially from those assumed in developing formula 72. For example, at the point of maximum bending moment, the shear is zero, and in consequence no web compression exists in the half of the girder between the neutral axis and the tension flange, while the web compression in the other portion of the girder is parallel to the flange and increases in intensity as the distance from the neutral axis increases. Between the end and the point of maximum moment the condition varies from that assumed in developing the formula to that just stated. While it is evident that the formula does not apply very closely to all these conditions, the fact that it gives a gradually increasing distance apart of the stiffeners as the shear diminishes is probably consistent with actual conditions.

The size of intermediate stiffeners cannot be determined theoretically; a good rule is to make the outstanding leg equal to or greater than two inches plus one-thirtieth the depth of girder. The other leg should be of sufficient width to permit of proper riveting.

In addition to the angles necessary to stiffen the web, vertical angles should be used at all points where concentrated loads of considerable magnitude are applied to the girder, in order to transmit these loads into the web without overstressing the flange rivets. The design of such stiffeners consists in selecting angles of sufficient area to withstand the load without crushing, and in determining the number of rivets necessary to transmit the load into the web the value of the rivet being limited either by bearing on the web or by double shear. Both types of stiffeners are indicated in Fig.72b.

For the results of experiments on the buckling of plate girder webs see article by Turneaure in the "Journal of the Western Society of Engineers for 1907. Vol. 12"

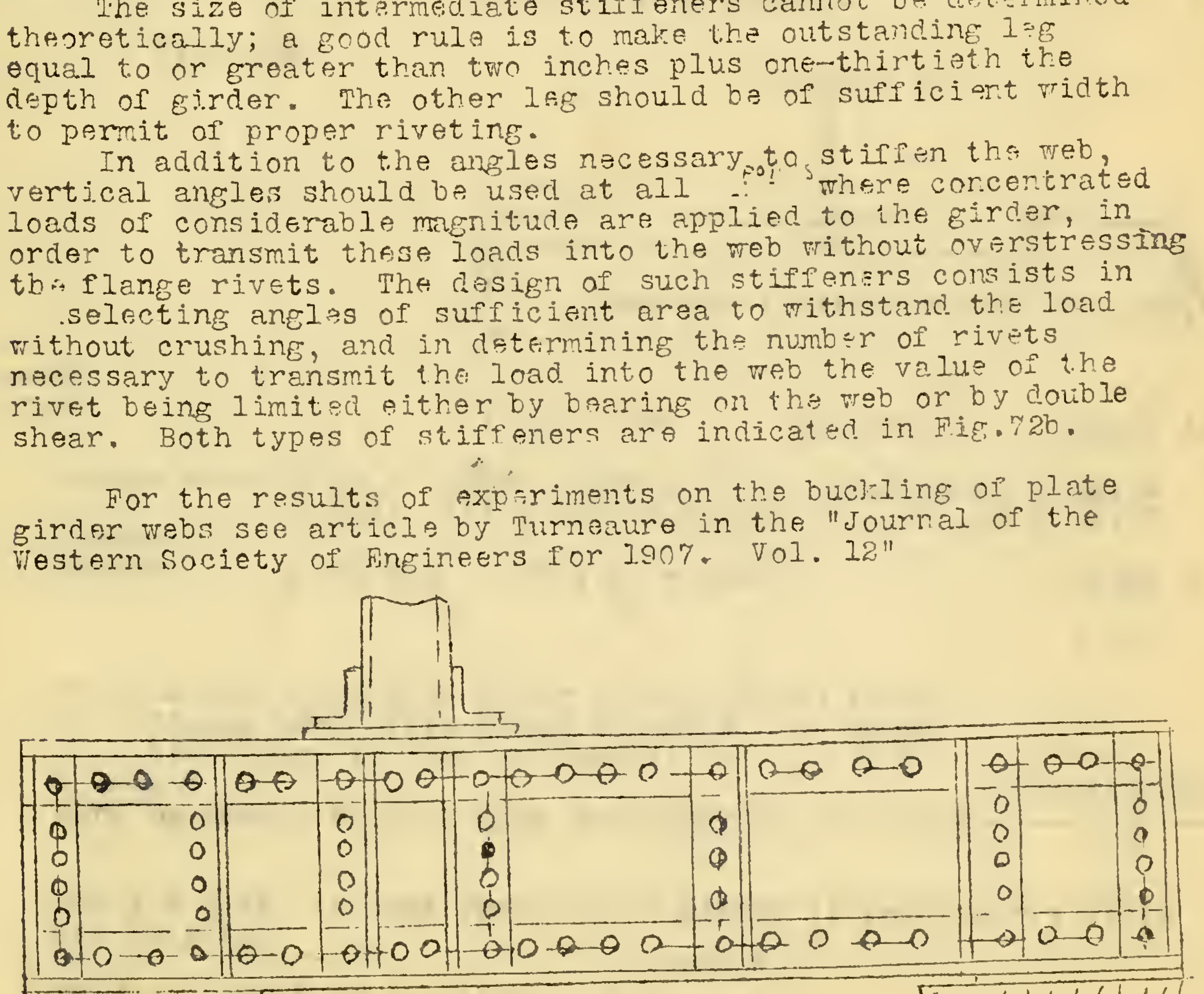


Fig 72b

Since the column lies at 45° to the axis no stiffeners will be needed if the value of d is found to be greater than the clear distance between flange angles. It is, however, customary to use stiffeners on all except smaller girders in order to stiffen the girder during fabrication and transportation and a common requirement is that the maximum clear distance between stiffeners shall be the depth of the plate between flange angles and shall not be greater than 6 feet.

For that portion of the girder where the bending moment is large and the shear relatively small the web conditions differ materially from those assumed in developing formula [illegible]. For example, at the point of maximum bending moment the shear is zero, and in consequence no compression exists in the half of the girder between the neutral axis and the tension flange, while the web compression in the other portion of the girder is parallel to the flange and increases in intensity as the distance from the neutral axis increases. Between the end and the point of maximum moment the conditions vary from those assumed in developing the formula in that [illegible] stated. While it is evident that the formula does not apply very closely in all these conditions, the fact that it calls for a gradually increasing spacing of the stiffeners as the moment diminishes is probably [illegible].

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For the results of experiments on the buckling of plate girder webs see article by Turneaure in the Journal of the Western Society of Engineers for 1907, Vol. 12.

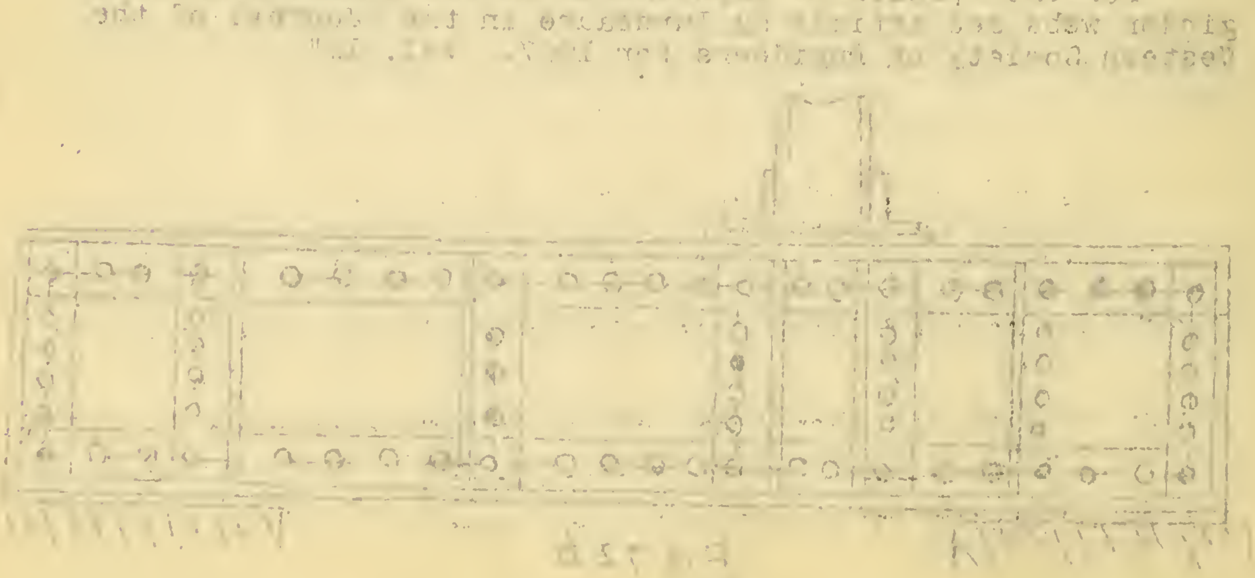


Fig. 12b

73 COVER PLATES. Cover plates are used to increase the flange area and to give a variable, and thus a more economical flange. It is not good design to use too many cover plates. In general the area of the plates should not exceed one-half the total flange area, unless the largest sized L's are used. As rivets should not be very long because of the difficulty of getting good rivets, the thickness of the metal in the flanges should not exceed 4 1/2 ". In case a greater flange area is required, use vertical flange plates as shown in Fig. 73a, or use a box girder.

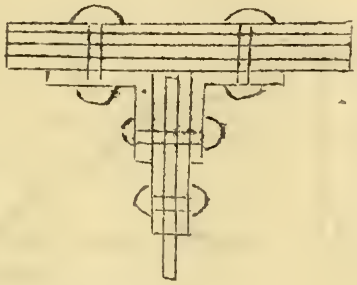


Fig. 73a.

To determine where a cover plate may end it is necessary to equate the bending moment which the girder can carry without the cover plate to the external bending moment at the end of the plate.

The following example serves to illustrate this method.

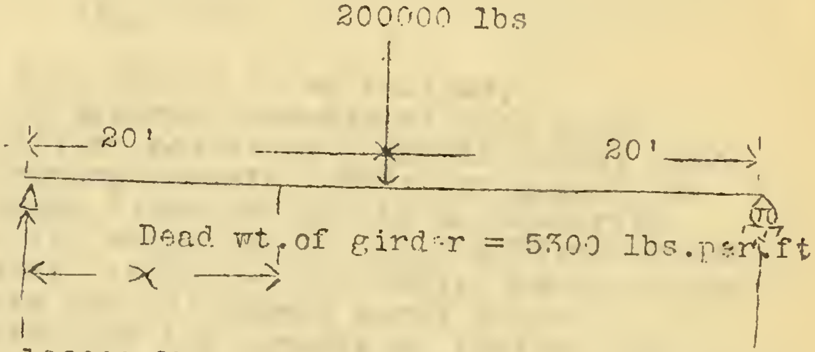


Fig. 73b.

Flange area (2 ∠s 6" x 6" x 5/8" @ 7.11 = 14.22-2.50 = 11.72
(2 plts. 16" x 1/2" @ 8.00 = 16.00-2.00 = 14.00

Tension flange $\frac{1}{12}$ web = $\frac{1}{12}$. 48 . $\frac{9}{16}$ = 2.25 = 2.25

27.97

To find end of outside cover plate proceed thus:-
Flange area after plate is cut = 20.97
Dist. c.g. to c.g. of flange = 47.4"

Bending moment which flange can carry in ft. lbs. = $\frac{20.97 \times 16000 \times 47}{12}$

Let x = dist. in feet from end of girder to point where plate can be cut.

Bending moment at x = $106000\,x - \frac{300\,x^2}{2}$

$$\therefore 106000\,x - \frac{300x^2}{2} = \frac{20.97 \times 16000 \times 47.4}{12}$$

Solve for x.

The actual length of the cover plate should be somewhat longer than the theoretical length in order that the stress may be properly carried into the plate. 12" extra is usually allowed at each end for this purpose.

For girder subjected to moving loads it is more advantageous to use a graphical method.

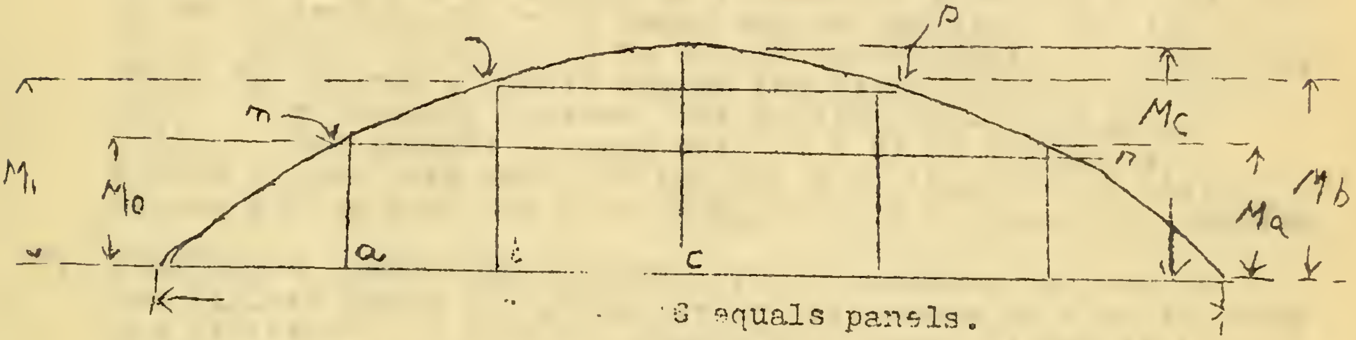


Fig. 73c.

The application of this method is as follows;:

Plot the span and the external moments at each panel point, connecting these latter points by a smooth curve, thus obtaining the curve of bending moments. (This curve is practically a series of straight lines and may be so used with safety, the influence of the weight of the girder being offset by the fact that a straight line curve for moving concentrated loads gives excess moments for all except panel points. (See Article 49). Compute also the moments of resistance of the girder, by the application of the common formula (See Article 63) without cover plates, with one cover, with two covers, etc. and plot these moments to the same scale as the external bending moments. Since the moment of resistance for each case is constant throughout the length of the girder it will be shown graphically by a straight line parallel to the girder axis.

The points of intersection of the straight lines representing the various moments of resistance with the curve of bending moments give the points where the cover plate may be cut.

This method is shown for a girder with two cover plates by Fig. 73b. M_a, M_b, and M_c are the external bending moments.

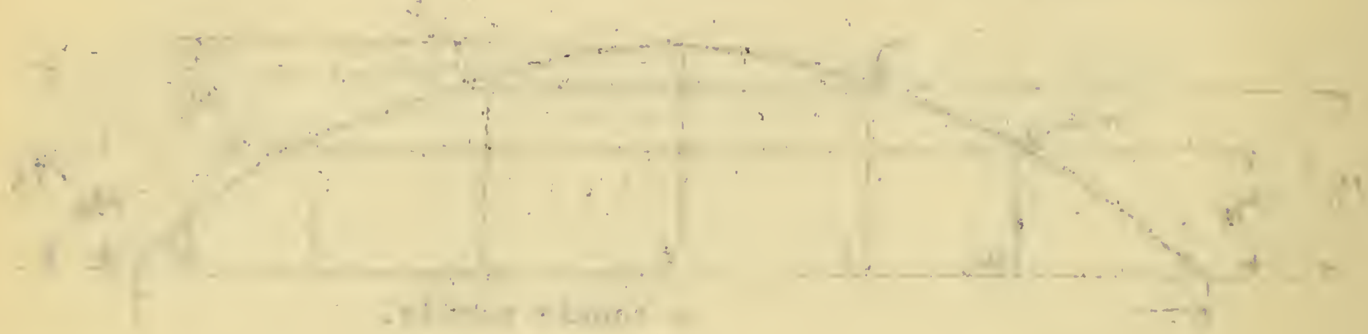
Mo is the moment of resistance without cover plates; M_1 with one cover plate. (The moment with two cover plates need not be plotted. The cover plate nearest the flange should extend from m to n, the outer cover plate from o to p. These are theoretical lengths and the actual plate should be made somewhat longer as previously stated.

[illegible]

[illegible]

[illegible]

[illegible]



[illegible]

The application of this method is as follows: [illegible]

[illegible] girder axis.

[illegible]

[illegible]

[illegible]

The flange width is an important feature and should be carefully considered in selecting angles and plates. It is common in railroad bridge practice to specify that the compression flange should be supported laterally at intervals not greater than 12 times its width, this being accomplished in half-through bridges by brackets attached to the floor beams and in deck spans by cross frames and horizontal bracing. In case it is necessary to deviate materially from this rule the flange should be figured as a column or the rule given in Article 59 for I beams may be applied. For the sake of appearance it is usual to select cover plates of sufficient width to project slightly beyond the flange angles on either side. They should, however, not project more than about 2 inches. For example flanges with 6" x 6" ∠s should have plates of not less than 13" and not more than 16" in width. Plates with a width in even inches should preferably be chosen.

74 CONNECTION ANGLES AND FILLERS. It is necessary either to use fillers under all plate girder stiffeners, or else to crimp the stiffener L's over the flange L's. There is but little difference in the cost of the two methods but the former is generally preferred.

One objection to the use of fillers is that unless the filler is riveted to the web plate by an independent row of rivets thus becoming practically a portion of the web, the rivets connecting the stiffener to the web are reduced in strength since they have to carry stress through the loose filler plate and thus are subjected to some bending. This is of no importance in intermediate stiffeners which serve merely to stiffen the web, but should be considered in the case of stiffeners carrying a concentrated load into the web. In such cases if loose fillers are employed an excess of rivets, say 50 percent, should be used.

The use of tight fillers is also advisable in some cases to increase the bearing value of rivets when they are limited by bearing on the web instead of by shear. The following example illustrates this;-

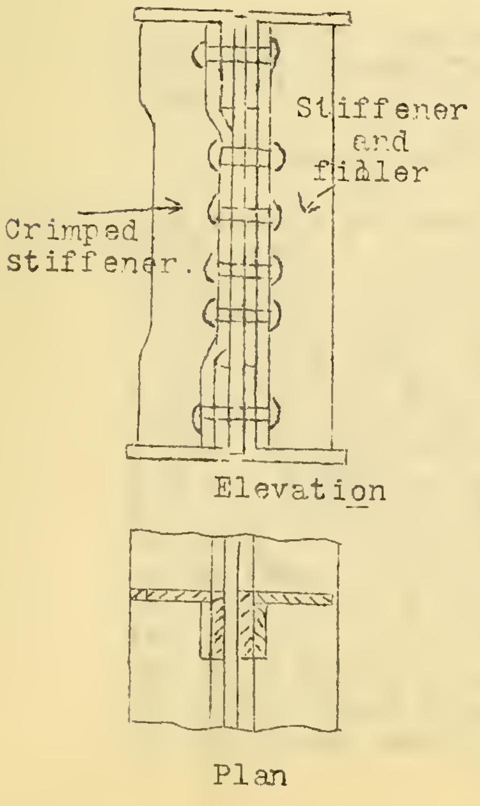


Fig. 74a.

[illegible]

[illegible]

[illegible]

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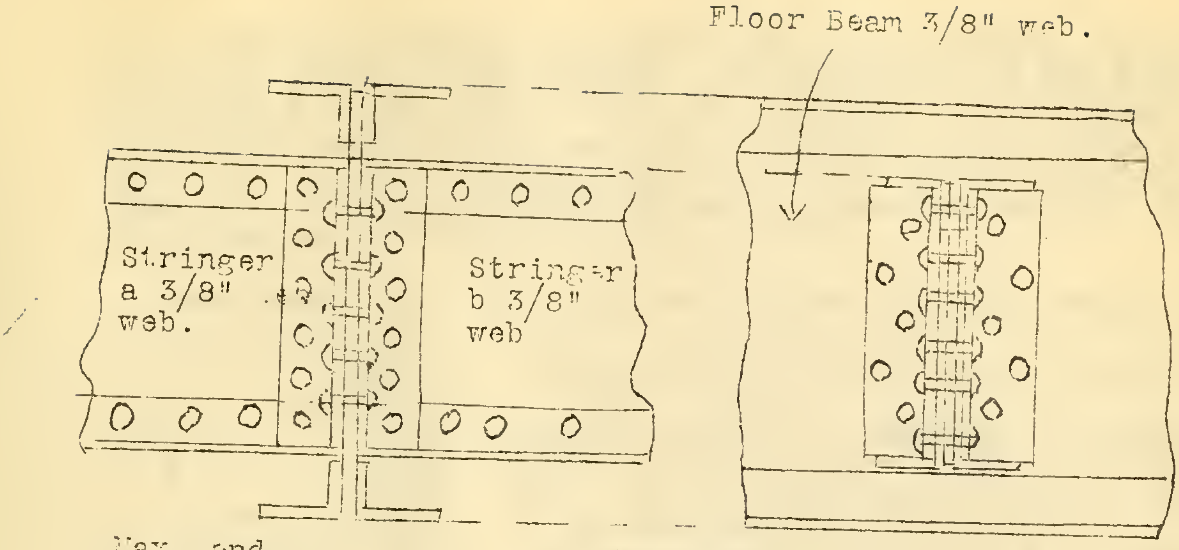


Max. end shear = 40000 lbs.

Max. end shear = 30000 lbs.

Fig. 74a.

Max. load from both stringers together upon floor beam = 65000 lbs. Allowable unit stress per square inch upon rivets =

	Bearing	Shear
Machine	24000	12000
Hand	20000	10000

For 7/8" rivets above units give the following values for 3/8" plates.

	Bearing	Shear
Machine	7875	7200
Hand	6560	6000

Assume that the rivets shown are all that can be used in the L's, (field rivets, which are hand rivets, are shown thus (•)

To carry 40000 # in stringer a, there are required $\frac{40000}{7875}$ = 6 rivets to connect the stiffener angles to the web. As indicated in the figure, the largest number of rivets that can be used is 6, but it is inadvisable to count upon those in the flange, and additional rivets should be added if the filler is to be a loose one, hence it is necessary either to use wider Ls with two rows of rivets or else a wide filler to increase the bearing value. The latter would be cheaper and consequently advisable. If the filler, therefore, be made wider and connected to the web by two extra rivets an additional stress equal to that which two rivets could carry can be taken from the web into the filler and by that carried into the rivets connecting the stiffeners. As these rivets would, however, have to carry a considerable bending moment in addition to the direct shear it is advisable to make a liberal allowance, hence it would be well in this case to use four rivets in the fillers placed directly opposite those in the stiffeners. The stringer would then be as shown in Fig. 74b.

One other point yet remains to be considered, viz., the shearing value of the rivets. The joint has so far been designed to permit the stress to be taken out of the web plate into the rivets; can the rivets carry this stress into the L's? As the thickness of the L's is not restricted they can be given sufficient bearing area but can the rivets carry the stress without shearing off?

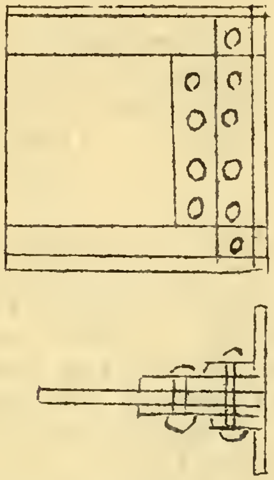


Fig. 74b.

As the rivets are in double shear there are needed $\frac{40000}{14400} = 3$ rivets; hence the number needed for bearing is sufficient for shear; otherwise it would be necessary, despite the wide filler, to use wider stiffener L's.
The connection of stringers to floor beam may be treated in a similar manner. 12 hand rivets are shown. These have to carry in single shear the max.shear in a single stringer, i. e., 40000 lbs. They also have to carry 65000 lbs in bearing upon the web.

$\frac{40000}{6000} < 12$ hence there are enough in shear

$\frac{65000}{6560} = 9\ (+)$ or 10, hence there are also enough in bearing.

75 WEB SPLICES. Owing to the limited length of plates which can be rolled it is frequently necessary to splice the webs of long and deep girders. The lengths obtainable are given in the different handbooks (See "Cambria Handbook," 1907 edition, pages 29 et seq.) and determine the location of the splices. Figs. 75a, 75b, and 75c show several methods of making such splices.

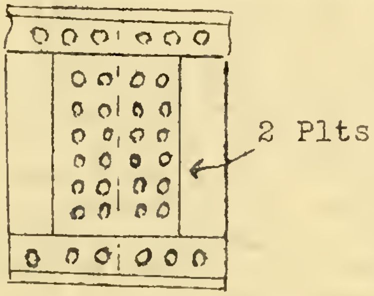


Fig. 75a.

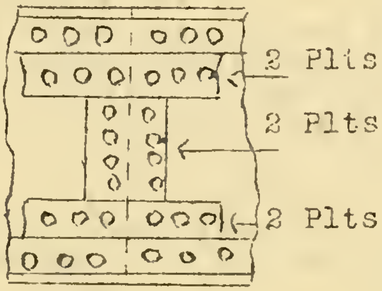


Fig. 75b

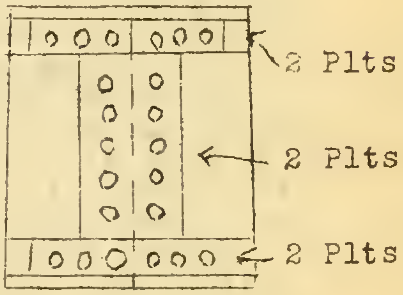


Fig. 75c.

Of these the type represents in Fig. 75a is best in appearance and is recommended for use.

In designing a web splice it is desirable to make the splice plates equal to the web both in shearing and bending strength, this being in accordance with the generally followed rule in good design, to make the details as strong as the number.

Since two splice plates should always be used, one on each side of the web, it is seldom necessary to make these plates thicker than the minimum allowable thickness, though in the case of girders with very thick webs, or of shallow girders with wide legged flange angles it is sometimes necessary to exceed this minimum. In such cases the thickness should be computed.

The computation for the splice shown in Fig. 75a of the number of rivets required to carry the combined shear and bending is more difficult. The method given in Article 67 is applicable but is less simple to apply, gives but slightly different results and is probably no more nearly correct than the following method.

In designing for this case assume the number and location of the rivets in one row and that the resistance of the rivet to torsion acts parallel to the axis of the girder, that is, that it acts in the same direction as the flange fibre stresses. Then compute the value of the rivets in torsion, and shear, and decide upon the number of rows needed. It will ordinarily be found that at least two rows are required, and that the rivets in each row must be spaced not more than 3" centre to centre.

In order to illustrate the method of computation the splice shown in Fig. 75d will be considered.

Assume the web of girder to be 61" x 7/16" and the angles 6" x 3-1/2" x 9/16". If 3/8" be the minimum allowable thickness of metal the minimum splice plates will be 54" x 3/8", and since the combined thickness of these is 3/4" which is much in excess of the 7/16" web, while the depth is but slightly less than that of the web, it is evident that the size of the plates will be sufficient. The arrangement of rivets must next be examined. The figure shows four rows in all with the rivets 3" centre to centre which is as near together as the rivets should be spaced. The next step is to find the bending moment which the web can bear. This may be found approximately and with sufficient accuracy for our purpose by taking the product of 1/12 the web area, the allowable unit stress, and the distance between the centres.

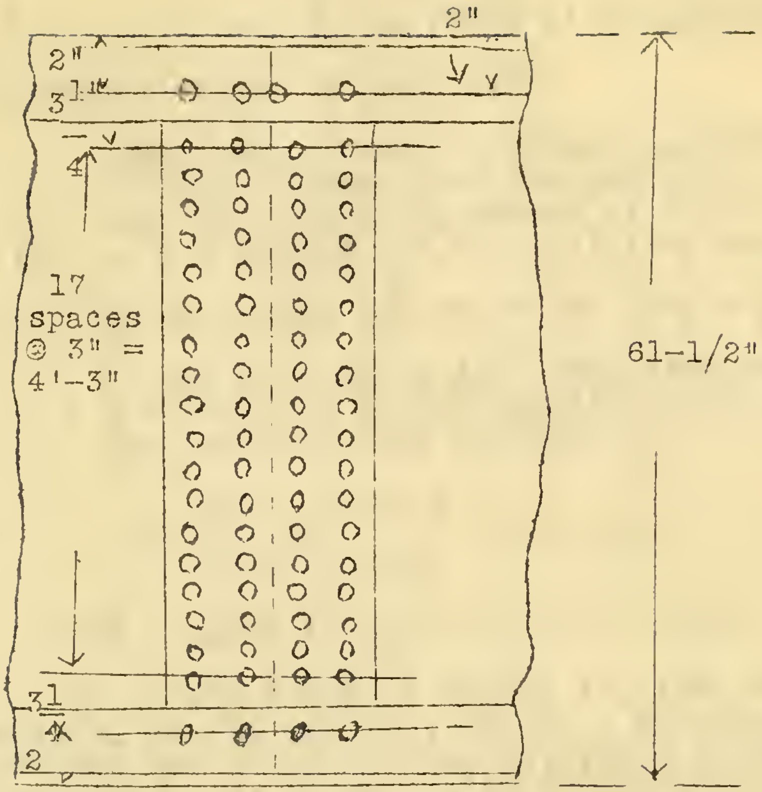


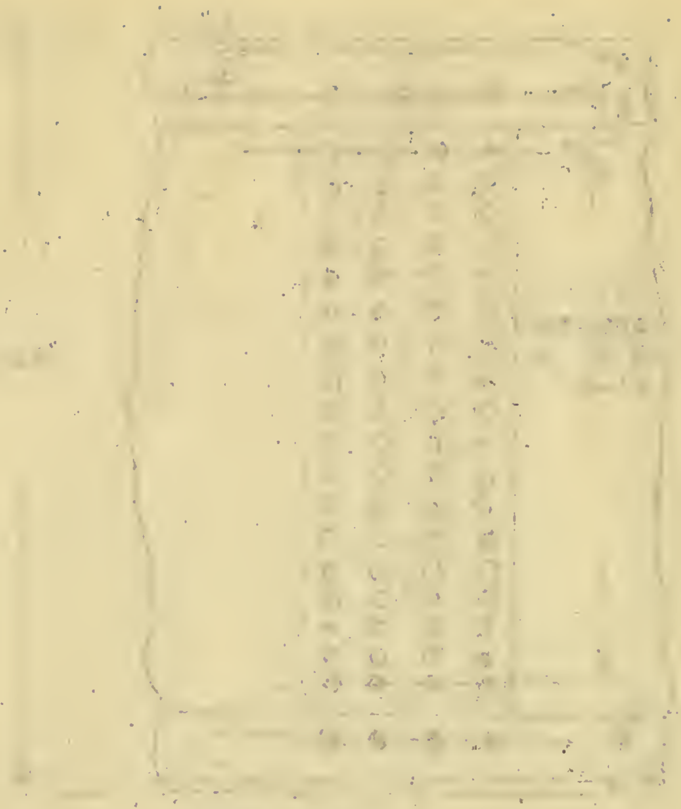
Fig. 75d.

of gravity of the flanges, this being merely the reversal of the method used in determining the proper allowance to make for the web in developing the formula for flange area. For the example under consideration the following equations will be obtained.

Bending resistance of web $= 1/12 . 61 . 7/16 \times (61\frac{1}{2} - 1\frac{3}{4}) \times f$

and if f, the allowable unit stress in bending be taken as 16000 #/" the bending resistance will equal 2,126,000 inch lbs. It is next necessary to determine the resisting moment of one row of rivets. Let the bearing value of the rivets be taken as 20000 and the shearing value as 10000 lbs. per square inch. The limiting value of the rivet is evidently its bearing value on the 7/16" web which equals 7/8 x 7/16 x 20000 = 7650. If the shear is assumed to be distributed uniformly over the web it may also be assumed as uniformly distributed over the splice rivets. The total shearing strength of the plate at 10000 lbs. per sq. inch is $61 \times \frac{7}{16} \times \frac{3}{4} \times 10000$ (allowing for reduction due to rivet holes) = 200000 lbs. If it be assumed that two rows of rivets are required on each side of the splice the stress per rivet will be $\frac{200000}{36} = 5550$ lbs.

The value of the rivet in bending will then equal $\sqrt{(7650)^2 - (5550)^2} = 5270$ lbs. It is clear, however, that only the top and bottom rivets will develop this stress, and that the stress which the other rivets may be counted upon as carrying will depend upon their distance from the neutral axis as



[illegible]

[illegible] elements.

[illegible]

[illegible]

[illegible]

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indicated in Fig. 75e in which the rivet at m carries $\left(\frac{X}{A}\right)5270$

As the resisting moment of each rivet

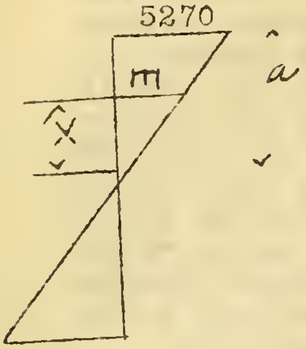

Fig. 75e.

equals its allowable stress multiplied by the distance from the neutral axis the resisting moment of the rivet at m = $\frac{5270X2}{a}$ and the resisting moment of an entire row of rivets equals $\frac{X^2}{a}(5270)$

For the case under consideration a = 25-1/2", ∴ the allowable moment which one row of rivets can bear =

$$2 \cdot \frac{5270}{25.5} \sum_{1-1/2}^{25-1/2} x^2 \text{ in. lbs.}$$

$= 2 X 20.7 X (25.5^2 + 22.5^2 + 19.5^2 + 16.5^2 + 13.5^2 + 10.5^2$

$7.5^2 + 4.5^2 + 1.5^2) = 414 X 2180 = 902500$ in. lbs. hence two rows will carry 1,815,000 in. lbs. which is somewhat less than the girder web was figured to carry, viz., 2,126,000 in. lbs., therefore the splice is not quite strong enough. The deficiency is, however, not large and is offset to some extent by the fact that the shear is not distributed uniformly over the web but is a maximum at the neutral axis, hence the rivets near the neutral axis may be assumed to carry more of the shear than the extreme rivets. Since these rivets have been counted upon to carry practically no bending moment this more accurate method would result in increasing the computed bending resistance of the rivets. To determine the exact distribution of the shear and the proper amount to be carried by each rivet is somewhat laborious and will not be attempted here but it should be noted that an increase of 500 lbs. in the allowable bending resistance of the outer rivets and a proportional increase in the other rivets would be sufficient to bring the moment of resistance of the two rows of rivets up to the required amount, and that this increase would require a decrease of only 300 lbs. in the amount of shear to be carried by the extreme rivets with a corresponding increase in the shear to be carried by the rivets near the neutral axis, and that such a change is a very reasonable one to assume.

Were the deficiency greater it would be necessary to use three rows of rivets which would reduce the shearing force per rivet and hence increase the resistance to bending, or to use a smaller rivet pitch so as to get more rivets in a row. It is undesirable, however, to use a smaller pitch than 3" and although 2 3/4" might be used the gain in bending moment would be little and it would seem more desirable to use three rows, or else give up the attempt to make the splice as strong in bending as the web, and instead place it at a point where the flange is not fully stressed and hence where the full bending resistance of the web need not be counted upon. The point where the excess in flange area is sufficient to make up for the deficiency in the strength of the splice

[illegible]

[illegible]

can be readily determined analytically in girders where the moment can be expressed analytically since it is only necessary to determine where the actual moment is less than the moment of resistance of the full cross section by an amount equal to or greater than the deficiency in bending resistance of the rivets, and to place the splice somewhere in this space. Where the moment can not be expressed analytically the position of the splice can be determined graphically by a similar method to that given in Article 73., that is, by drawing curves to show the actual moment and the moment of resistance. In applying this to the case in hand the moment of resistance should be made equal to the moment of resistance of the full section minus the deficiency in the moment of resistance of the splice rivets. Since for such girders the curve of moments would ordinarily be drawn in order that the cover plate lengths may be determined the application of the method would require only the addition to the figure of an extra horizontal line.

The above method is for a splice which will be as strong in bending and shear as the web. It should be stated, however, that frequently the web has considerable excess strength in shear at the point where it has to be spliced, and that in consequence it is not absolutely necessary to make the splice as strong in shear as the web since if it is strong enough to carry the actual shear it will do. As a general rule, however, it is desirable to make the detail as strong as the member, and this rule should be adhered to unless good reason exists for a deviation from it.

76 FLANGE ANGLE SPLICES. In very long girders it is frequently necessary to splice the flange angles. When this has to be done only one angle in each flange should be spliced at a section. A common practice is to splice the top angle on one side of the girder, and the bottom angle on the other side at a section a little to one side of the centre of the girder, and to reverse this process for a corresponding section on the other side of the centre. The splice should always be made by another angle the cross section area of which should be equal to that of the angle to be spliced. In order to simplify the construction the splice angles for the tension flange should be exactly like those for the compression flange, hence the net area of the splice angle should equal the net area of the main angle.

In order to obtain a splice of neat appearance and which answers the above requirements it is usually necessary to select an angle with the same width of legs as the main angle but $\frac{1}{16}$" or $\frac{1}{8}$" thicker, and plane off the projecting legs so that they may be flush with the main angle.

The following example illustrated this;-Determine the splice angle required for a 6" X 6" X $\frac{1}{2}$" flange angle. The net area of the main angle = 5.75 - 1.00 = 4.75 sq. in.

The net area of a 6" x 6" x $\frac{9}{16}$" angle planed to fit the 6" x 6" x $\frac{1}{2}$" angle = 61.44 - 1.12 - $2(\frac{9}{16} \times \frac{1}{2})$ = 4.76 sq. in. hence this angle has just the right area and should be used.

[illegible]

[illegible]

[illegible]

The following example [illegible]

[illegible]

This particular [illegible] the rough area and should be made.

Fig. 76 shows by cross hatching the portion of the angle to be cut off. The outer corner must also be rounded off to fit the fillet of the main angle.

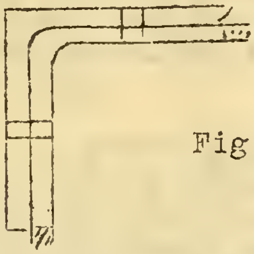

Fig. 76a.

The number of rivets required in the splice angle may be determined, if there are no cover plates, by computing the stress which the angle can bear and dividing it by the value of one rivet, the strength of the rivet being generally limited by single shear. If the angles are equal-legged one half the number of rivets needed should be used in each leg. If the legs are unequal each leg should have its proportional part of the total rivets required; e.g. if a 6" x 4" angle is to be spliced and if 20 rivets are needed in all put $\frac{6}{10}$ 20= 12 rivets in the 6" leg, and $\frac{4}{10}$ 20 = 8 rivets in the 4" leg.

Cover plates are generally required in girders the flange angles of which have to be spliced and in such girders the number of rivets needed may be somewhat in excess of the number required to carry the total stress which the angles are good for, since the increment in stress in a distance equal to the length of the splice angle must be taken care of. Referring to Fig. 76b it is evident that the rivets at a-a must carry from the flange into the splice angle in the distance m n one half the increment in flange stress in that distance (the other half going through the same rivet to the flange angle on the left hand side) plus one half the stress in the main angle at m (since the angle is equal-legged). The rivets at c should be computed to carry the same amount since it is proper to assume that all the increment in flange stress is carried by the cover plates the angle being fully stressed before cover plates are added. The rivets at both a and c are

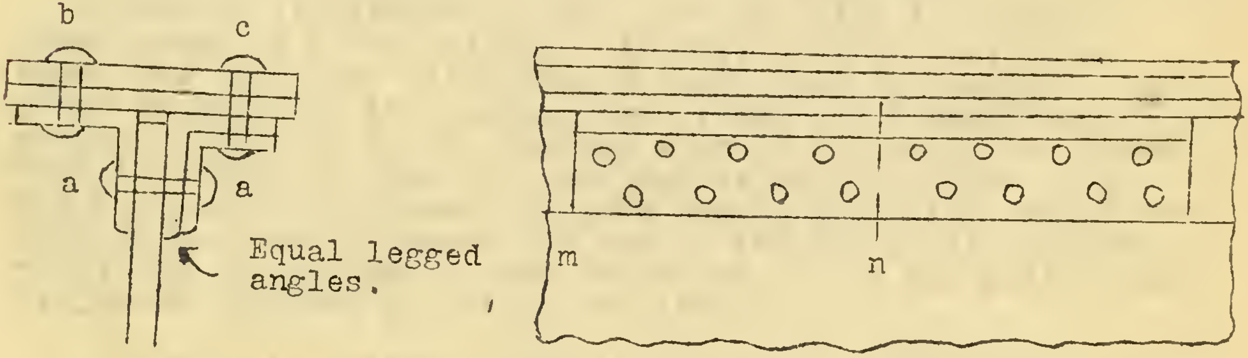

Fig. 76b.

limited usually by single shear, and should be designed accordingly. Since the splice would ordinarily be placed near the centre of the girder where the increment in flange stress is small, it is usually sufficient to determine the number of

rivets required to splice the angle assuming it to be stressed to its full value and to add one or two rivets to carry the flange stress increment. If no cover plates be needed it is unnecessary to consider the increment in stress since if the splice rivets be determined for the full value of the angle this stress is surely equal to the stress in the angle at m plus the increment in mn.

77 COVER PLATE SPLICE. A cover plate splice may always be made by the addition of a splice plate of the same size as the plate to be cut, and the use of sufficient rivets to transmit the full stress from one plate to another, with the addition of a liberal percentage of extra rivets, say 33% for each plate intervening between the plate to be spliced and the splice plate. Such a splice is shown in Fig. 77a.

The disadvantage of long rivets subjected perhaps to bending moment because of the intermediate plates together with the unsightly appearance of such a splice makes it desirable, if the girder has more than one cover plate to splice one cover plate by means of another. This may be done by properly choosing the section where the splice is to be made.

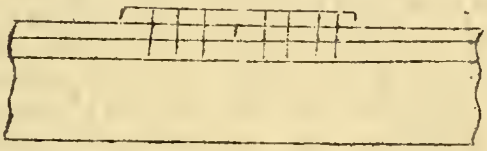

Fig. 77a.

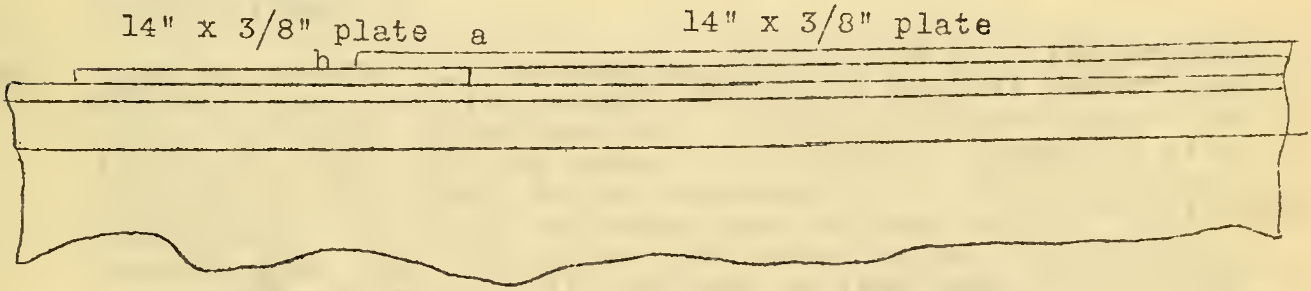

Fig. 77b.

This is illustrated by Fig. 77b in which the lower cover plate is to be spliced. If this plate be cut at a where the top cover plate should begin, and if the top plate be extended to b making the distance a b such that enough rivets can be put between a and b to carry the stress that the plate is good for, the splice will be made. If the top plate be thinner than the bottom plate the splice would have to be located nearer the end of the girder at a point where one cover plate of the thickness of the top plate would be just sufficient to carry the stress.

It will be observed that this method is based upon the transfer of all the stress from the end section of the plate to be spliced into the plate immediately above it. The intermediate section of the spliced plate instead of the upper plat will then take the additional increment of flange stress.

CHAPTER V.

TRUSSES.

78 TRUSSES DEFINED. A truss is a structure composed of a number of separate pieces connected at their intersections only. The connection is sometimes made by plates to which the members are riveted, such a truss being called a riveted truss, or sometimes less properly a riveted girder; or else by large bolts or pins the latter type receiving the name of pin truss. The points of intersection of the members are called the joints, and the outer forces should be applied so far as is possible at these joints only. This is accomplished by the use of floor beams or in a roof by purlins. As the depth of plate girders is limited by the available width of plates and by the inability to ship by rail single pieces wider than than 10'-6" or thereabouts, it is necessary to use trusses where greater depths are desired for stiffness or economy. The common practice in the United States at the present time is to use beams or girders up to lengths of 90' or 100'; riveted trusses above these lengths up to 150' or 175'; and pin trusses above this length.The use of shorter pin truss spans for railroad bridges has been given up because of lack of rigidity and consequent early wearing out of the bridge. See figure in Chapter I for illustration.

79 CLASSIFICATION. All trusses may be divided into two general classes based upon the methods necessary for the determination of the stresses in the members; if these stresses can be determined by statics the truss is statically determined with respect to the inner forces; otherwise it is statically undetermined. It should be noted that a truss may be statically undetermined with respect to the outer forces, and yet be statically determined with respect to the inner forces, and *vice versa*. The former is usually the case with draw bridges, the latter with the double intersection trusses frequently used in simple span bridges.

80 THEORY. The theory upon which the computation of truss stresses is based assumes that the members are connected at the intersections of the centre of gravity lines by frictionless pins and that in consequence the stresses are direct stresses. That this deviates considerably from the truth for riveted trusses is evident, the error in pin trusses is less but not negligible hence the common theory of trusses is by no means an exact one. The secondary stresses produced by resistance to motion at the joints are, however, small in well designed trusses, as compared with the primary stresses, and the theory based upon

this assumption has stood the test of many years, is in general use, and its approximations are sufficiently allowed for in all ordinary structures by the factor of safety.

81 METHODS. The methods necessary for the computation of the stresses in statically determined trusses are very simple, and consist merely of the application of the three equations of equilibrium to portions of the truss, these portions being chosen in such a way as to enable the stress in a given bar or bars to be immediately found. There are in common use three methods of accomplishing this result; the method of joints, the method of moments, and the method of shears. All of these are applications of the general method and differ only in detail. In the computation of a truss it is often advantageous to employ all three methods choosing for each bar that which is best adapted to it. The method of joints is the most general of these methods and will be considered first.

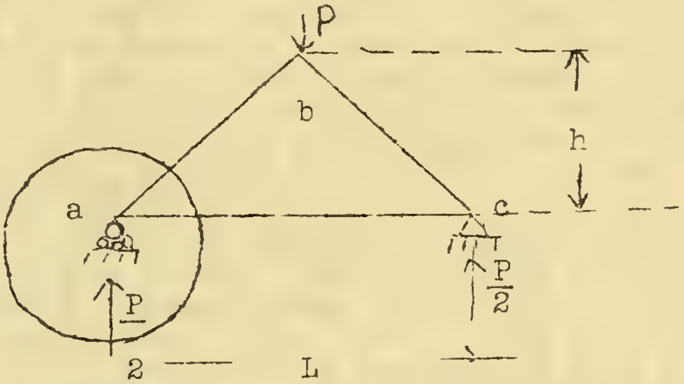

Fig. 82a.

82 ANALYTICAL METHOD OF JOINTS DESCRIBED.

Fig. 82a represents a simple truss carrying a load at the apex. Let a section be taken around the joint at a and the remainder of the truss removed. As the entire truss is in equilibrium the portion enclosed by the circular section must also be in equilibrium and the problem resolves itself into that of determining the forces in the bars consistent with equilibrium of particular portions of the truss cut off by similar sections taken at a sufficient number of joints to permit the determination of all the unknown stresses. Fig. 82b shows the condition which exists, assuming that the stresses in the bars are axial stresses this being in accordance with the general theory. It should be carefully observed that this method deals with the stresses in the bars rather than with the bars themselves.

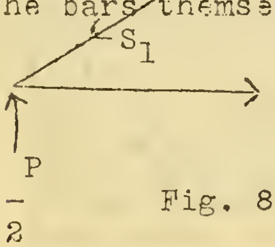

Fig. 82b.

Referring to Fig. 82b it is evident that as there are but two unknown forces, S_1 and S_2, the two equations of equilibrium, $\Sigma H = 0$ and $\Sigma V = 0$ will be just sufficient to determine these, and since all the forces meet at a point the equation $\Sigma M = 0$ will be satisfied by any value of S_1 and S_2 and

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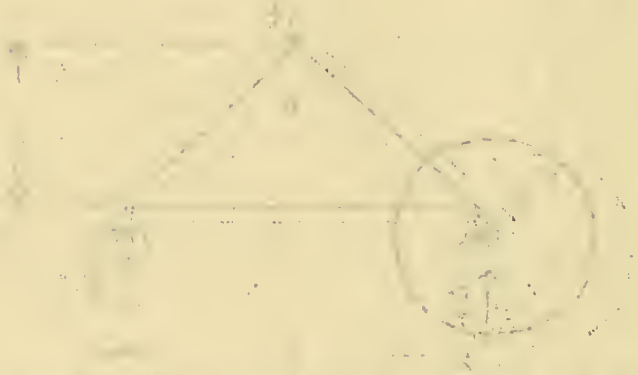

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need not be considered. With the stresses in bars ab and ac thus determined a section may next be taken at either joint b or joint c, and the stress in bar bc be computed in a similar manner thus completing the computation for this truss.

83 CHARACTER OF STRESS. The determination of the character of the stress is often more important than that of its magnitude, as a bar designed for tension may fail if the stress is compression even if its magnitude is small. To determine its character it is sufficient to arbitrarily assume the direction of the stresses before applying the equations of equilibrium. If the solution gives a positive result for a stress it shows that this stress acts in the direction originally assumed.

In this connection it should be carefully observed that the internal stresses in a tension bar continually tend to shorten the bar, that is to pull the ends together, hence tension in a bar always acts away from the joints at both ends, and compression towards the joints. Fig. 82b illustrates this. S_2 is shown acting away from the joint, that is, in tension; S_1 on the other hand is assumed in compression and is shown acting towards the joint. If the joint at the top of the truss should next be investigated it would be necessary to show S_1 as acting towards that joint also, since the computations give a positive value for S_1 thus indicating that it acts in the assumed direction.

It is safer for the beginner to assume the stress in all bars as tension or away from the joint. Positive values will then indicate tension and negative values compression. This is in accordance with the common but not universal convention of representing tension (which increases the length of a bar) by a plus sign.

84 DETERMINATE AND INDETERMINATE STRUCTURES. For the truss under consideration there are three unknown bars. The stresses in two of them have been determined by considering one joint only; the stress in the other may be found by taking either of the other joints. Since by taking both of the other joints there would be four equations and only one other unknown bar it would seem as if there were too many equations. This is not correct, however, as these equations must suffice to determine the unknown reactions as well as the unknown bars; since equilibrium of each joint involves equilibrium of the entire structure, that is, for this particular structure and in general for all structures which are statically determined with respect to the reactions there must be three more equations than there are bars. In other words the 6 equations of joints for this truss are not independent but are related in such a manne as to satisfy the 3 general equations of equilibrium for the truss as a whole, viz: $\Sigma X = 0$, $\Sigma Y = 0$ and $\Sigma M = 0$ which may for most cases be replaced by the more common equations.

$$\Sigma H = 0 \qquad \Sigma V = 0 \qquad \Sigma M = 0$$

There are therefore for truss shown in Fig. 82a but 3 independent equations which can be used in determining the bar stresses, hence these stresses are determinate.

In general it may be said for all trusses which are sta

cally determined with respect to the outer forces, that if n equals the number of joints, 2n - 3 equals the number of bars which the structure must have to be determinate. If it has more bars the stresses can not be computed by statics; if less it will not be rigid and will collapse except under special conditions.

If it be desired to build a structure which because of the number of points of support or for other reasons ordinarily would be statically undetermined with respect to the outer forces, it may be possible to make the structure determinate in all respects by properly choosing the number of members. For example, in the case of a cantilever truss, a diagonal over a pier is sometimes omitted for this reason. In swing spans also diagonals are often omitted or made very small in order to reduce the numbers of unknowns. If the unknown components of the reactions be four it is evident that there can be only 2n - 4 bars if the structure is to be made determinate.

85 MODE OF PROCEDURE. ANALYTICAL METHOD OF JOINTS. In the solution of problems by the method of joints the following mode of procedure should always be adopted:

1 - Compute reactions.
2 - Select a joint at which only two bars meet.
3 - Assume the stresses in these bars to be tension, that is to act away from the joint; and apply the equations of equilibrium. If the stress in either bar is found to be negative it indicates that the bar is in compression instead of tension.

4 - Consider any other joint at which only two unknown bars meet and determine the stresses in these bars in the same manner and proceed thus until all the stresses have been determined.

86 APPLICATION OF ANALYTICAL METHOD OF JOINTS. The following numerical example has been worked out to show the application of this method.

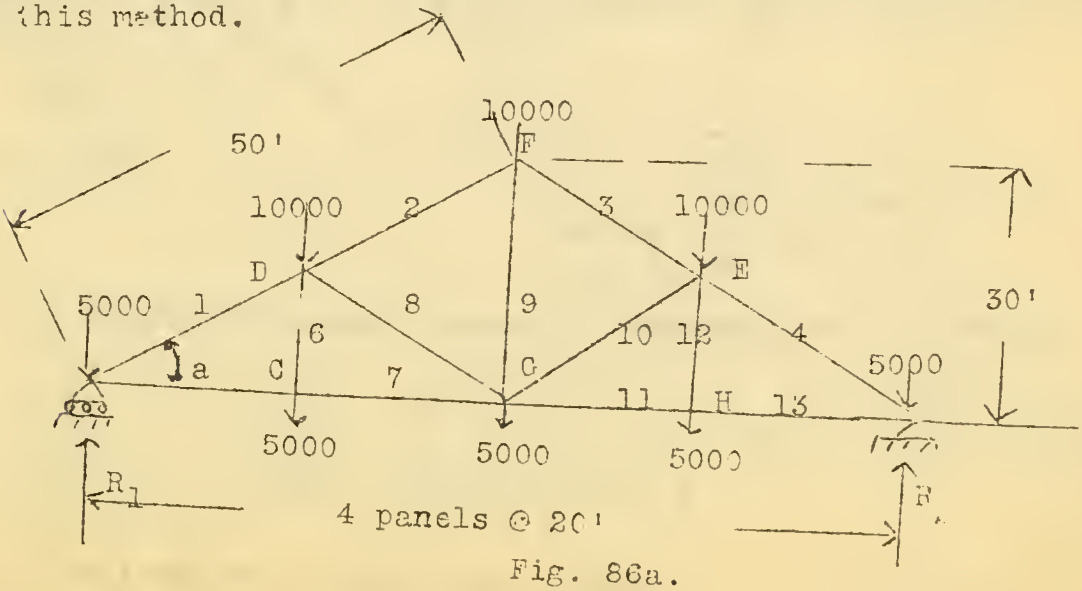

Fig. 86a.

$$R_1 = R_2 = \frac{55000}{2} = 27500$$

Joint B.

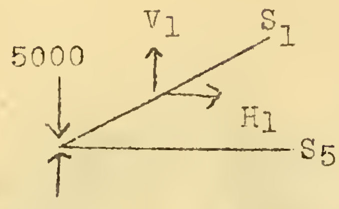

Fig. 86b.

$\underline{\Sigma V = 0}$ $V_1 + 27500 - 5000 = 0$

$V_1 = -22500$

$$H_1 = \frac{40}{30} V_1 = (-)\ 30000$$

$$S_1 = \frac{50}{30} V_1 = 37500\ (-)$$

$\underline{\Sigma H = 0}$

$H_1 + S_5 = 0$

$-30000 = -S_5$

$S_5 = 30000\ (+)$

Joint C.

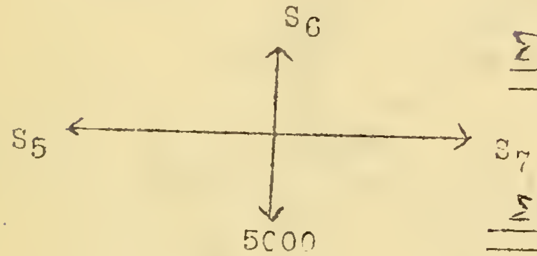

Fig. 86c.

$\underline{\Sigma V = 0}$ $S_6 - 5000 = 0$

$S_6 = 5000\ (+)$

$\underline{\Sigma H = 0}$

$30000 - S_7 = 0$

$S_7 = 30000\ (+)$

Joint D.

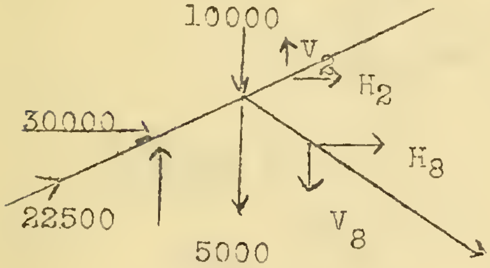

Fig. 86d.

$\underline{\Sigma V = 0}$

$22500 - 15000 + V_2 - V_8 = 0$

$\underline{\Sigma H = 0}$ $30000 + H_2 + H_8 = 0$

$$\text{But } H_8 = \frac{20}{15} V_8 = \frac{4}{3} V_8$$

$$\text{And } H_2 = \frac{40}{30} V_2 = \frac{4}{3} V_2$$

∴ from above equations may be obtained the following:

$$7500 + V_2 - V_8 = 0$$

$$30000 + \frac{4}{3} V_2 + \frac{4}{3} V_8 = 0$$

Solving $V_2 = -15000$ $H_2 = -20000$

$$V_8 = -7500 \quad H_8 = -10000$$

$$\therefore S_2 = 15000 \times 5/3 = 25000\ (-)$$

$$\text{and } S_8 = 7500 \times 25/15 = 12500\ (-)$$

The computation for a bar such as 8 may sometimes be advantageously referred to other than horizontal and vertical axes. If in this particular case the X axis be taken along the upper chord BF, and the y axis perpendicular to it the condition at the joint will be as shown in Fig. 86e. It is clear that in this case the value of Y_8 is given at once by the equation $\Sigma Y = 0$, and equals (-) 9000.

The actual stress in the

$$\text{bar} = \frac{12000}{\sin \theta}$$

$$= \frac{12000}{2 \sin x \cos X} \quad \text{(See Fig. 86a)}$$

$$= \frac{12000 \times 25}{24} = 12500\ (-)$$

It will be noticed that the stress in this case has been determined without reference to stresses S_1 and S_2, and is a direct function of the stress in bar C and the panel load at D.

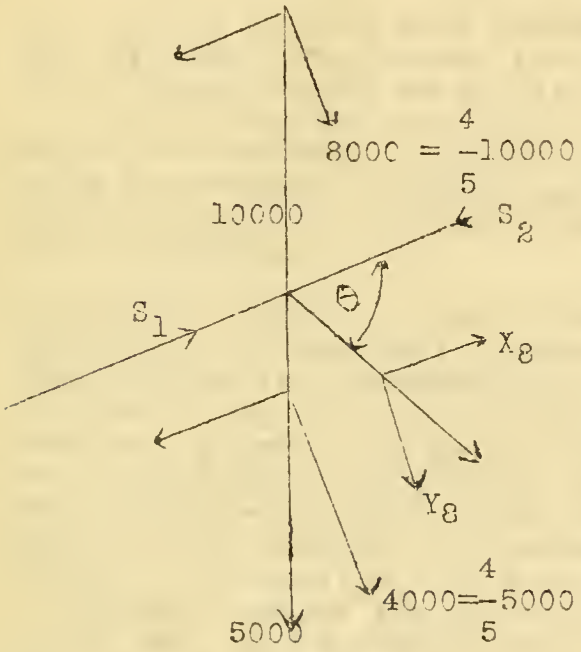

Fig. 86e.

Joint F

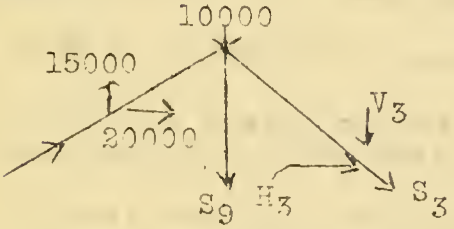

Fig. 86f.

$$\underline{\Sigma V = 0} \quad 15000 - 10000 - V_3 - S_9 = 0$$

$$\underline{\Sigma H = 0} \quad 20000 + H_3 = 0$$

$$H_3 = 20000\ (-)$$

$$\text{and } V_3 = 3/4\ H_3 = -15000$$

$$S_9 = 20000\ (+)$$

$$\text{and } S_3 = 15000 \times 5/3 = 25000\ (-)$$

Since the truss is symmetrical and the loads are also symmetrical, the stresses in the bars of one half the truss are identical with these in the bars of the other half, hence further computations are unnecessary.

As a check consider joint G which will have forces acting as shown in Fig. 86g.

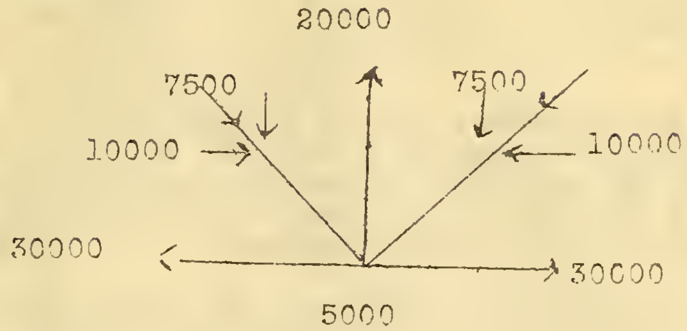

Fig. 86g.

It is evident that these forces are in equilibrium hence the stresses are checked to a certain extent. In practice further checks should be applied.

It should be especially noted that (–) signs in all cases where the unknown stresses have been assumed as tension show that compression actually exists, and that in all truss problems hereafter considered tension will be considered as (+) and compression (--).

87 GRAPHICAL METHOD OF JOINTS DESCRIBED. The analytical method just given is perfectly general but too laborious to use in determining the stresses in all the bars of an ordinary truss, though it may be used with great advantage for certain specific members. A graphical method based upon the same principles is well adapted for many types of trusses, particularly roof trusses with non-parallel chords, and should be thoroughly understood. This method consists of drawing polygons of forces for each joint in successi_n, the polygon being so combined as to considerably reduce the labor which would be required if each joint were to be considered separately. The stresses in the bars can be obtained by scaling the sides of the polygons. Like other graphical processes this method is less precise than analytical methods and errors in scaling the stresses are easily made and difficult of detection. The closure of the figure, however, indicated that no error of importance has been made in the graphical work.

88 MODE OF PROCEDURE - GRAPHICAL METHOD OF JOINTS.

1 Draw a sketch of the structure to any suitable scale and show on it all the outer forces including reactions.

2 Designate all the forces and bars by letters so located that each force and each bar will lie between two letters and only two. For illustration of this see Fig. 89a.

3 Draw a polygon of outer forces. This should be drawn to a scale of sufficient size to give the desired accuracy.

The forces should be plotted in the order in which they are reached by going around the figure in a clockwise direction, and should be lettered at the ends by the letters in the order obtained by this clockwise rotation. This polygon should close if the reactions have been correctly determined.

4 Draw triangle of forces for all the joints beginning at any joint where an outer force and two bars only meet and proceeding thence by joint selecting the joints in such an order that at no joint will there be more than two undetermined forces to consider.
The sides of these triangles which represent the outer forces should be the sides of the force polygon. The sides representing bar stresses should be lettered at the ends by the letters obtained by going around the joints in a clockwise direction. The diagram thus drawn should form a closed figure.

5 Determine the magnitude and character of the bar stresses from the diagram. The magnitude of the stress in any member equals the length of the line of the diagram parallel to the bar in question measured to the scale of the force polygon, its character is determined by the order in which the letters are reached in going about a joint in a clockwise direction. For example to determine the character of the stress in bar CI of Fig. 89a note that CI in the stress diagram, Fig. 89b, acts downward to the left as determined by the order in which the letters are reached in going around joint (2), hence the stress in CI also acts downward to the left, or, towards the joint, since the bar is above the joint, and is, therefore, compression. A similar result is obtained by considering joint 4. For this joint clockwise reading gives the designation of the bar as IC and IC in the stress diagram acts upward to the right, that is towards joint 4, since the bar is below this joint.

The example which follows represents clearly the application of this method and shows by the closure of the diagram that no error of importance has been made in the graphical work.

89 APPLICATION OF GRAPHICAL METHOD OF JOINTS.

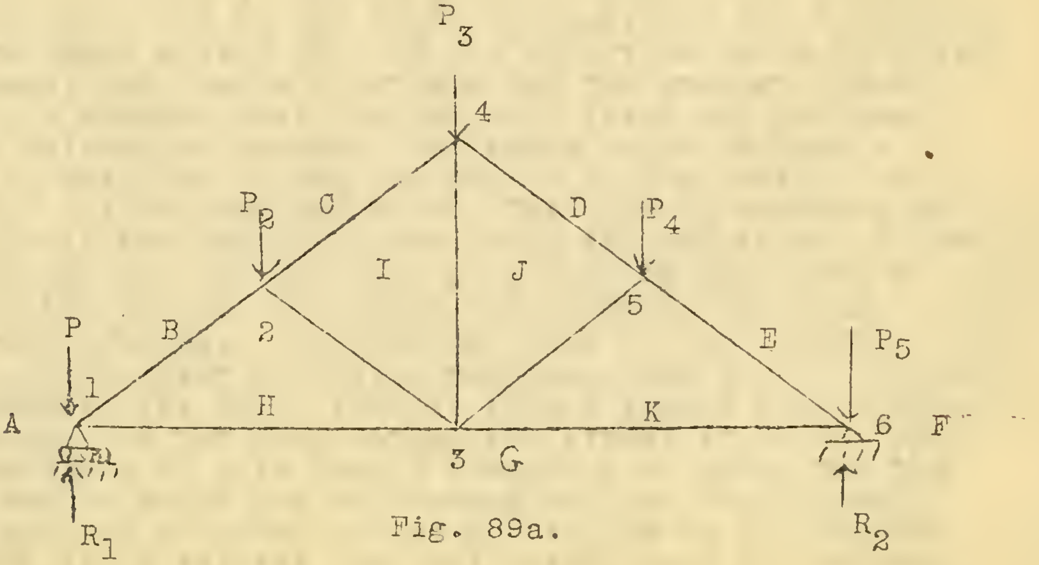

Fig. 89a.

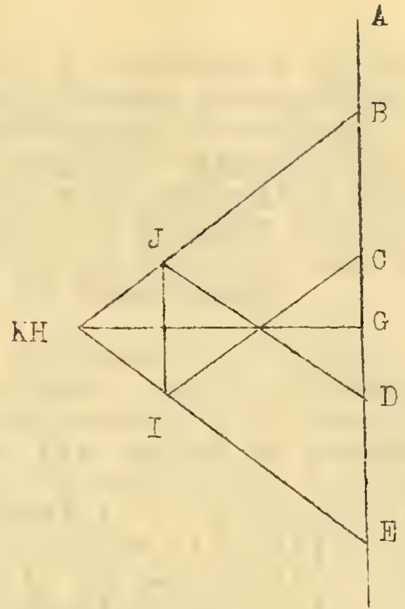

Fig. 89b.

Fig. 89a shows the structure drawn to scale and with all the outer forces represented in direction and point of application. The force polygon is ABCDEFGA in Fig. 89b which is a straight line since all the forces are vertical In it AB = P_1, BC = P_2, etc. The reactions R_1 and R_2 represented by GA and FG may be determined analytically., or graphically by methods given later.

The triangle of forces is first drawn for joint (1). The forces which act at this joint are R_1, P_1, the stress in bar BH, and the stress in bar HG, and these forces must be in equilibrium. Of these P_1 and R_1 are known in magnitude and direction and their resultant equals GB. BH and HG are known in direction but not in magnitude, hence there are but two unknown quantities, and it is evident that the value of these may be found by drawing a polygon of forces. The figure GABHG is such a polygon and is obtained by drawing from B a line parallel to BH, and from G a line parallel to GH. The line HB measured to the same scale as the force polygon gives the magnitude of the stress in the bar BH, and the line HG gives the magnitude of the stress in the bar HG. It remain to determine the character of these stresses. Considering joint (1) and reading around it in a clockwise direction starting with B, BH is found to act downward to the left, that is from B toward H thus showing compression. In the same manner the stress in GH is found to be tension since it acts from H towards G or away from the joint. This method would not be correct had not the external forces been plotted by going around the figure in a clockwise direction, but it is evident that this being done the method is correct; since in order to have GA, AB, BH, and HG in equilibrium the stresses in BH and HG must act as stated.

The next joint to be considered is joint (2) since there are now but two unknown forces acting there and they can, therefore, both be determined. To obtain them draw C I and H I in the force polygon parallel respectively to the corresponding bars in the truss; they will intersect at I. CI acts towards joint (2), and IH also acts towards the joint hence compression occurs in both these bars. In a similar manner the stresses in the other bars may be determined.

90 INDETERMINATE CASES. The method of joints, graphical or analytical, is perfectly general and applicable to all trusses, but in order to apply it successfully to some types of trusses, it is necessary to choose the method of procedure with care. For example in solving by the analytical method the truss shown in Fig. 90a, it is not possible to consider the joints in succession beginning at the abutment, but after solving the bars BL, CM, LM, MN, LK and NK it is desirable to determine PQ, doing this by applying the equations of equilibrium referred to axes along the top chord and at right angles to it, and then to determine PO using as axes the bar OR and a line at right angles to that. It will then be possible to figure the stress in the undetermined bars of that half of the truss.

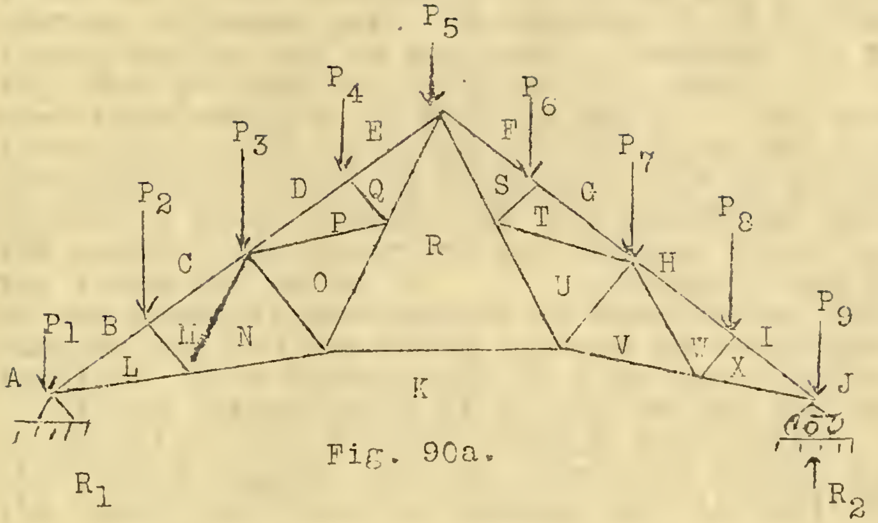

Fig. 90a.

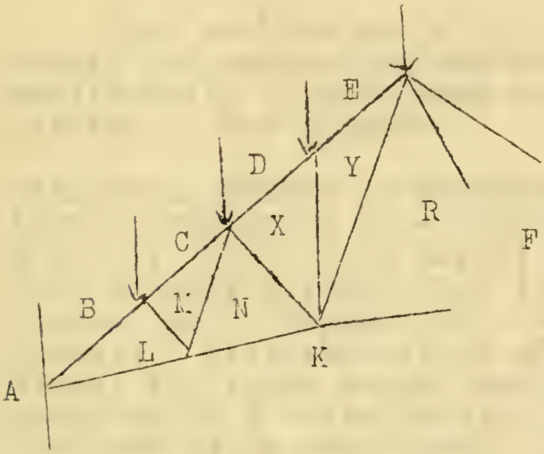

Fig. 90b.

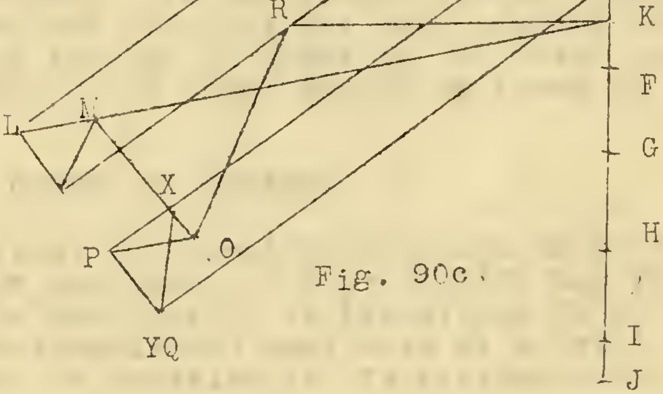

Fig. 90c.

In the graphical solution of this structure a similar difficulty also arises. After the stresses in bars BL, CM, KL, LM, MN, and NK and the corresponding bars in the other half of the truss have been determined no joint exists at which only two unknown stresses act. To overcome this difficulty the following device may be employed:– Consider the truss shown in Fig. 90b in which bars PQ, QR, RO, and OP of the original truss have been replaced by bars xy and yr. The truss is still determinate since one joint and two bars have been eliminated. Moreover the stress in Ey equals that in EQ since if the stress in EQ be computed by the analytical method of joints in the manner just described but working from the right end of the truss its value is clearly seen to be independent of any possible arrangement of bars on the left. The stress diagram for the new truss may now be continued and the point y located. This corresponds to Q of the original truss hence P is at the intersection of QP and DP and the remainder of the construction may be made without difficulty. Fig. 90C shows the stress diagram for the left half of the truss. That for the right half would be similar and is omitted.

This problem may also be solved by a combination of graphical and analytical methods, the stresses being determined analytically in such bars as are necessary and those values plotted in the diagram.

91 ANALYTICAL METHOD OF MOMENTS DESCRIBED. This method of finding truss stresses is based upon the application of the equation, $\Sigma M = 0$. It is very useful for determining stresses in special bars of many stresses but is not so general as the method of joints and is inapplicable to many of the bars of the simplest trusses. Like the method of joints it also is a method of sections, the truss being considered as divided into one or more portions by a cross section and the equilibrium of one of these portions being considered. It can be applied to determine the stress in a given bar when all the undetermined bars cut by the section except the one in question, or their prolongations, meet at a point which point should be taken as the origin of moments.

92 MODE OF PROCEDURE. METHOD OF MOMENTS.

1 Divide the truss by a section so drawn as to separate the truss into two or more parts. This section may be straight or curved but all the bars cut by it the stress in which is unknown (or their prolongations) must meet at a point which should not be the bar in question or its prolongation.

2 Apply the equation of moments using the point of intersection described under 1 as the origin and considering that one of the portions of the truss the simplest equation. The equation must include the moment of all the outer forces acting on the portion of the truss under consideration, together with the moment of the unknown bar stress which should be assumed as tension, Clockwise moments should be considered as positive.The section is commonly taken as cutting but three

bars two of which meet at a point while the third is the bar under consideration. It is sometimes simpler to deal with the moments of the components than with that of the forces themselves, particularly when the force may be resolved at a point such that the lever arms of one of the components is zero.

3 Solve the equation for the unknown stress. A positive result shows that the bar is in tension.

93 METHOD OF MOMENTS. APPLICATION. The application of this method is clearly illustrated by the following numerical example.

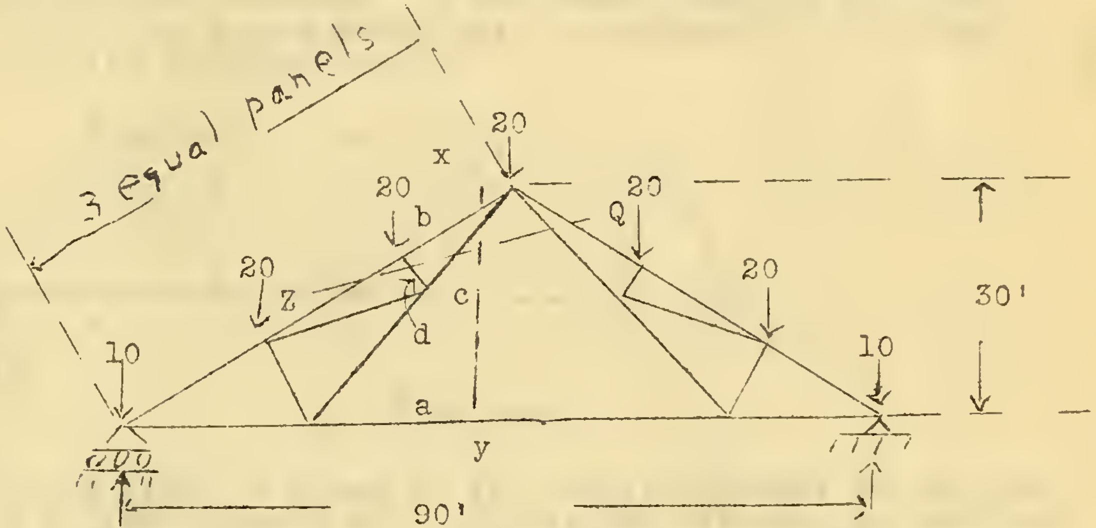

Fig. 93a.

Bar a. In this case the section xy fulfils the required conditions; that is, it cuts three bars two of which meet at a point, and the other is the bar a. If that portion of the truss to the left of the section be now considered, it is evident that it will be held in equilibrium by the outerforces and the stresses in bars a, b, and c, and that the moment of all these forces about any point in the plane of the truss must equal zero. If moments be taken about the intersection of b and c, the moment of the stresses in these bars will be zero, hence the equation of moments will include only the known outer forces and the unknown stresses in bar a which can in consequence be readily computed. The equation will be as below, assuming stress in a as tension.

$$(60 - 10)\,45 - 20 \times (15 + 30) - 30\,S_a = 0$$

$$S_a = (+)\,45$$

Using the same section the stress in bar b may be found by taking the origin of moments at the intersection of bars a and c

In above cases the section xy was assumed to be vertical or nearly so; this, however, is not necessary, and a horizontal or inclined section may be used provided that the bars cut by

[illegible]

[illegible]

(c) [illegible]

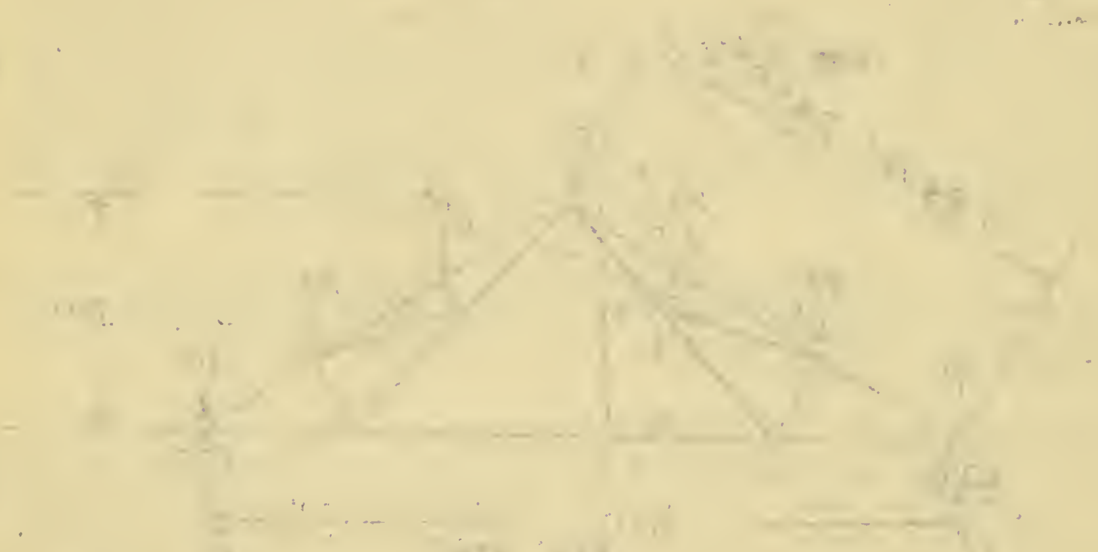

Fig. 176.

[illegible]

$$[illegible] = 0$$

[illegible]

it fulfil the stated conditions. For example the stress in bar d may be computed by this method using the section ZQ and taking moments about the apex of the truss. This gives the following equation:

$$20 \times 15 + S_d \times 18 = 0 \text{ whence } S_d = -20 \times 15/18.$$

It should be noted that the method of moments is inapplicable to the determination of the stresses in the web members of a parallel chord truss such as a somple bridge truss since in such trusses the sections through the web members would have to cut the chord bars which would meet at infinity and the equations thus derived would be indeterminate.

94 METHOD OF SHEARS DESCRIBED. This is another special method which can often be used to great advantage in the determination of the stresses in certain bars and particularly in diagonals of parallel chord trusses. In the truss shown in Fig. 94a it is clear thst if the truss acts as assumed in the theory, that is if the stresses in all the bars are axial, a shear on the section xy must be carried entirely by the diagonal ad . . . since the stresses in the bars ab and cd are horizontal, and the application of the equation $\Sigma V = 0$ to the portion of the truss on the left of the section xy necessitates the equating of the shear on the section with the vertical component in the diagonal. This is illustrated by Fig. 94b in which by applying $\Sigma V = 0$ we obtain

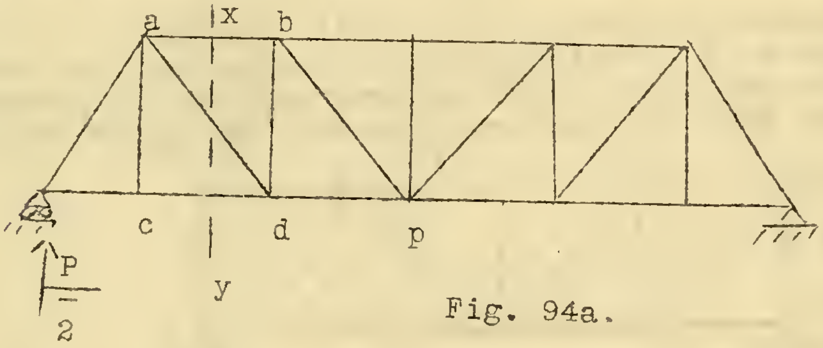

Fig. 94a.

$$\frac{P}{2} - V_3 = 0 \text{ whence}$$

$$V_3 = \frac{P}{2}$$

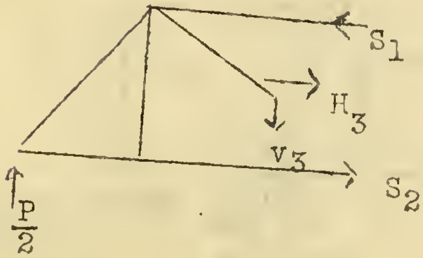

Fig. 94b.

[illegible] equation:

[illegible]

[illegible]

[illegible]

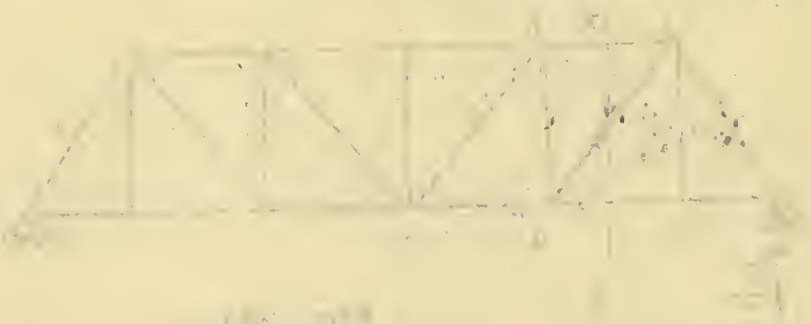

Fig. [illegible]

[illegible]

[illegible]

Fig. [illegible]

95 MODE OF PROCEDURE—METHOD OF SHEARS.

1°. Divide the truss into two parts by a vertical section passing through the bar in question. This section may cut any number of bars provided all are horizontal except the one under consideration, but in general it should be so chosen as to cut not more than three bars.

2°. Apply the equation $\Sigma V = 0$, to that portion of the truss which has the fewest outer forces acting upon it. The equation should include the vertical components of all the outer forces acting upon the portion of the truss selected, and the vertical component of the unknown bar stresses which should be assumed as tension.

3°. Solve the equation thus obtained for the unknown vertical component. A positive result shows that the stress in the bar is of the character assumed in writing the equation.

96 METHOD OF SHEARS, APPLICATION. The accompanying example clearly illustrates the application of this method to the determination of the stresses in the web members of a simple bridge truss carrying uniform dead and live loads.

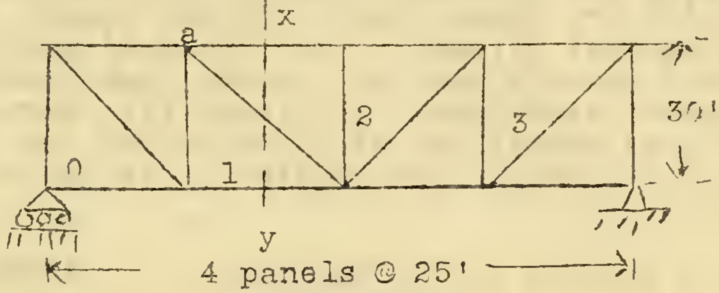

Fig. 96a.

Let the dead load be taken as 1000 lbs. per foot all on the bottom chord, and the live load as 2000 lbs. per ft. also on the bottom chord. The panel loads will then be 25000 lbs. dead and 50000 lbs live, and the dead shear on the section xy will be (+) 12500 lbs. To get the max. live shear on xy assume full panel loads at points 2 and 3, and no load at point 1. This gives a live shear in the panel of (+) 37500 lbs. The vertical component in the bar ab will then be (+) 12500 dead and (+) 37500 live, and with the vertical component known the actual stress can be easily computed. The forces acting upon that portion of the truss to the left of the section with this loading will be as shown in Fig. 96b from which it is readily seen that by assuming the stress in the diagonal to be tension and applying the equation $\Sigma V = 0$, V_1 is found to act down as assumed. Had the truss been inclined instead of horizontal the proper course to pursue would have been to resolve the vertical forces into normal and tangential components, and apply $\Sigma V = 0$ to the normal forces.

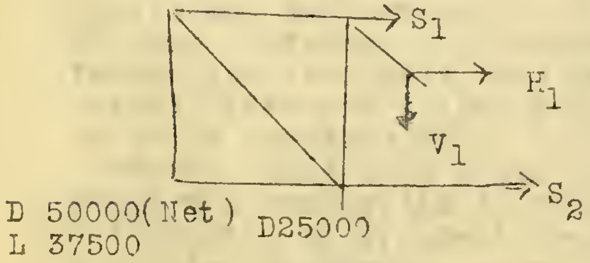

Fig. 96b.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

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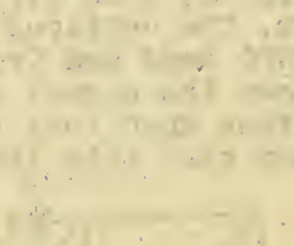
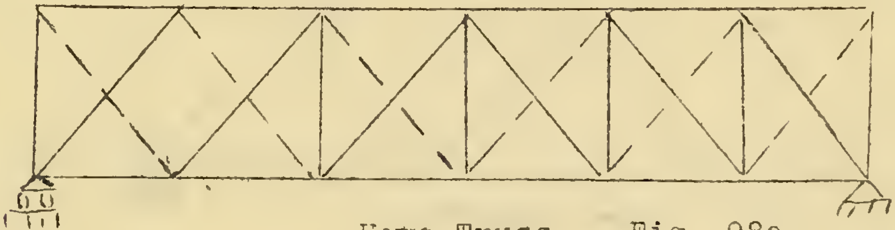


[illegible]

97 GENERAL RULES. The student should note carefully that the three methods which have been explained; viz: the method of joints, the method of moments, and the method of shears, are all methods of sections, and that in their application it is always necessary to assume a section through the truss and write an equation of equilibrium between the outer forces acting upon the truss on one side of the section, and the forces in the bars cut by the section. In computing a bar by analytical methods the first step is to determine the method to use. It should next be decided where to take the section and what portion of the truss to consider. Finally the proper equations should be applied between all the outer forces acting on the portion selected and the stresses in the bars cut.

A combination of the three methods which have been explained, joints, moments, and shears, enables us to compute readily the stress in any or all members of a statically determined truss. In order, however, to figure the stresses in the simplest manner, it may be necessary to study with considerable care some of the members, in order to determine which method should be adopted. In bridges, however, the forms of trusses which are in common use for simple spans are not numerous and the best methods to adopt can be readily learned by the study of these conventional types. For roof trusses the graphical method of joints will usually be found most convenient though sometimes it may be desirable to supplement this method by computing the stress in certain bars by one of the other methods.

98 TYPES OF TRUSSES. The forms of bridge trusses most frequently adopted are given in this article.



Howe Truss. Fig. 98a.

The Howe truss is usually built with chords, diagonals and end verticals of wood, and intermediate verticals of iron. Stresses in diagonals will be compression, and in intermediate verticals tension. The diagonal members shown by the dotted lines are called counters, and are needed in every panel in which negative shear can be obtained from any loading whatsoever, since such shear would produce tension in the main diagonal the connection of which to the other members is usually ill adapted to transmit tension. Otdinarily in such trusses a counter is used in every panel though not needed to carry shear, its size being made one-half that of the main diagonal; that is, if two equal sticks are needed for the main diagonal, the counter would be, one stick of the same size.

[illegible]

[illegible]

[illegible]

[illegible]

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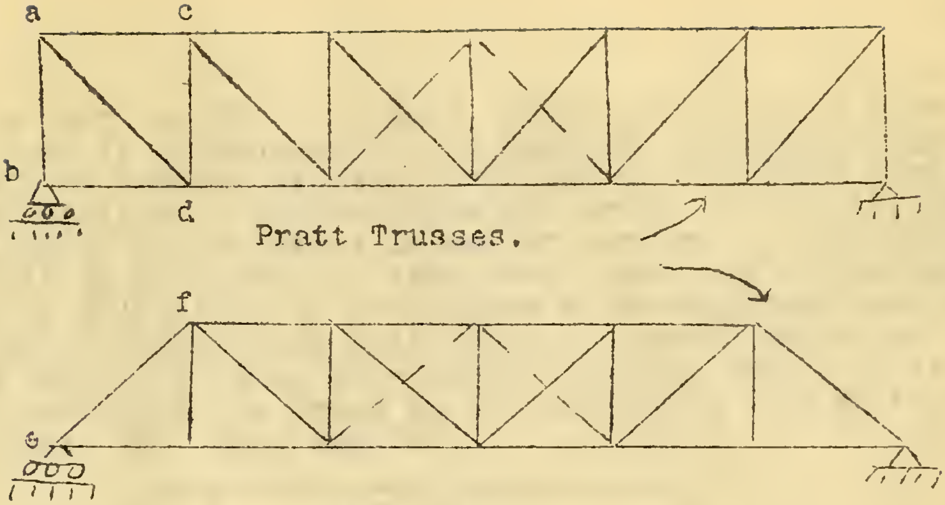


Fig. 98b.

The Pratt truss is the most common type of bridge truss. It is usually built of steel, and has tension diagonals and compression verticals. The truss with end verticals is not commonly employed for through bridges since it is less economical of material than the other form in which the compression members ab and ac and the tension member ad, are replaced by the one compression member ef. In the Pratt truss counters are used only in those panels in which negative shear may occur, and are used in such panels, only for economy, it being less expensive to add an extra tension bar of small size, than to make the main diagonal of sufficient strength to carry both tension and compression. In heavy railroad bridge construction it is becoming more and more common to avoid light counters and to make members subject to reversal of stress as compression pieces.

The Warren truss is very commonly adopted for riveted trusses of small span. No counters are used and the diagonals in panels where negative shear can occur are made compression members. It is, however, evident from the arrangement of the diagonals that every other one would have to be a compression member to withstand the positive shear.

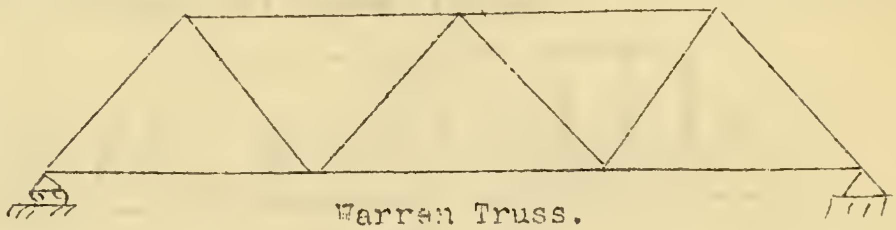
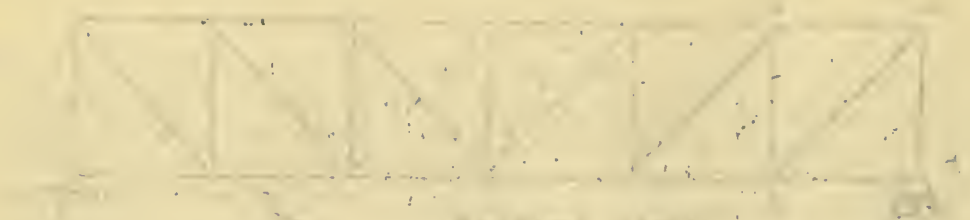


Fig. 78c.

In all cases in order to obtain economy of material it is essential that the ratio of depth of truss to span should be within certain limits, which are approximately from 1/5 to 1/8 the span and that the diagonals should make angles of approximately 45° with the horizontal.To obtain both of these



Pratt Truss.

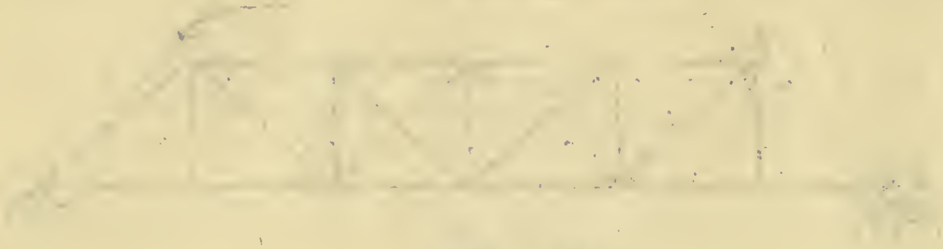
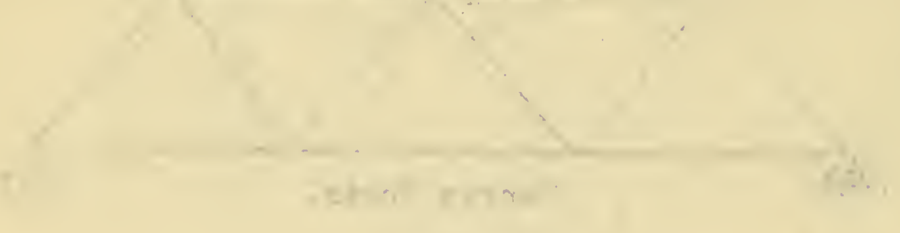


Fig. [illegible]

The Pratt truss is the most common type of bridge [illegible] It is usually built [illegible] verticals and com- [illegible] [illegible] compression [illegible] [illegible] [illegible] members.

The Warren truss [illegible] commonly used for [illegible] (because of small span). [illegible] used for the diagonals.

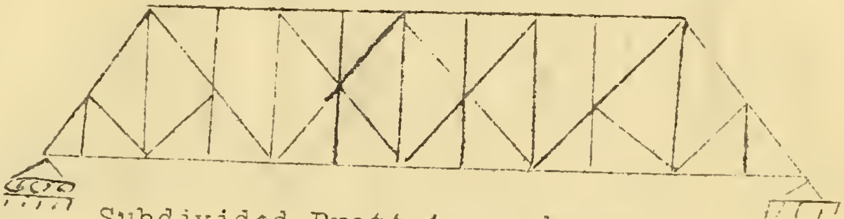


Warren Truss.

Fig. [illegible]

[illegible] diagonals [illegible] compression [illegible]

[illegible] suggested that the ratio of depth of truss to span [illegible] results it is clear that the panel length should vary with the span. As it is undesirable to use very long panels on account of the bending stresses produced in the chord bars by their own weight when greater than 25' or 30' in length, and because also of the increase in weight per foot of the stringers as their span increases, the panel length is seldom made in excess of 35' though in some spans of recent construction panel lengths greater than this have been used. In order to obtain panels of reasonable length in long spans, it is common to subdivide the truss by a secondary system as indicated in Figs. 98d, 98c, and 98f.



Subdivided Pratt truss known as the Baltimore truss.

Fig. 98d.

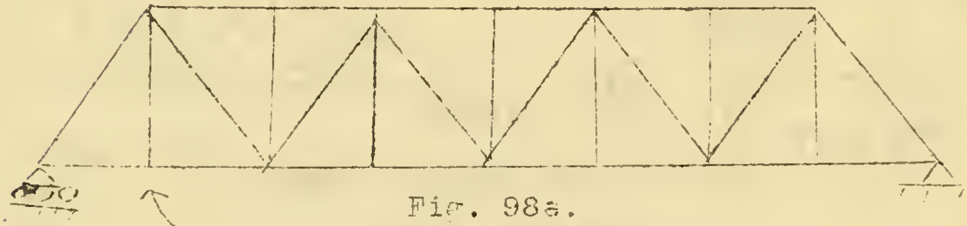


Fig. 98e.
Subdivided Warren Trusses.

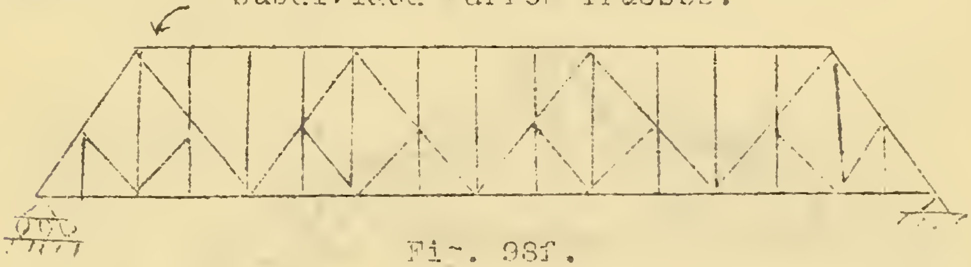
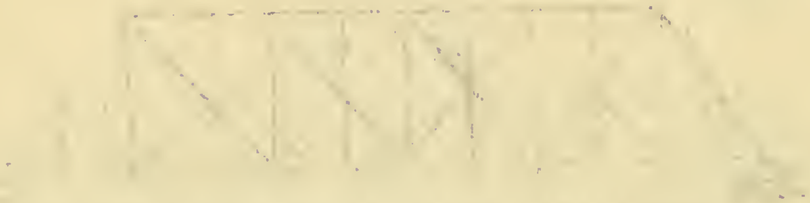


Fig. 98f.

For very long spans it is usually more economical to make the truss deeper at the centre than at the ends. If the depth is increased in proportion to the increase in moment it is evident that the chord stress would remain essentially constant throughout the entire length of the span, and that the chord would in consequence be much lighter at the centre than it would were the end depth to be continued throughout the span. The stresses in the diagonals would be increased in such a case, but the net result would be a saving of material, hence if minimum weight alone were to be the governing element, it would be desirable to make all trusses of varying height.

[illegible]



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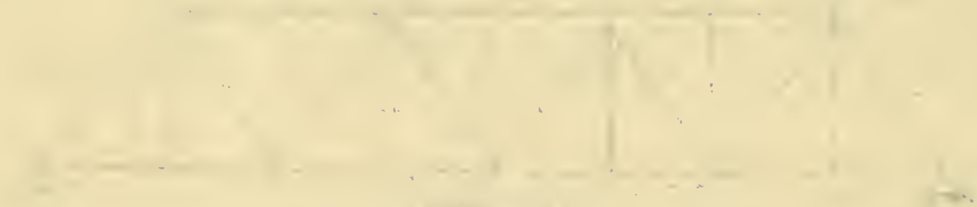

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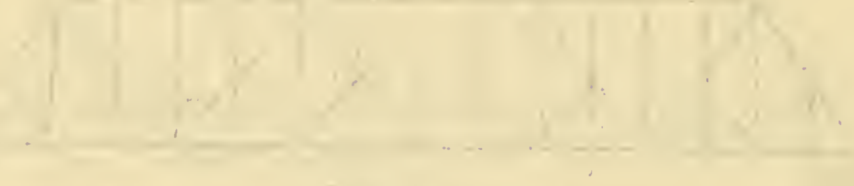

[illegible]

[illegible]

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It is necessary, however, to consider also economy of labor, and since trusses of varying depth are more expensive of labor it is evident that they should be used only for structures in which the savings of weight balances or exceeds the increased cost of construction. This point is usually reached only in spans of considerable length, say 300' and over, and the type of truss commonly used in such spans is shown below.

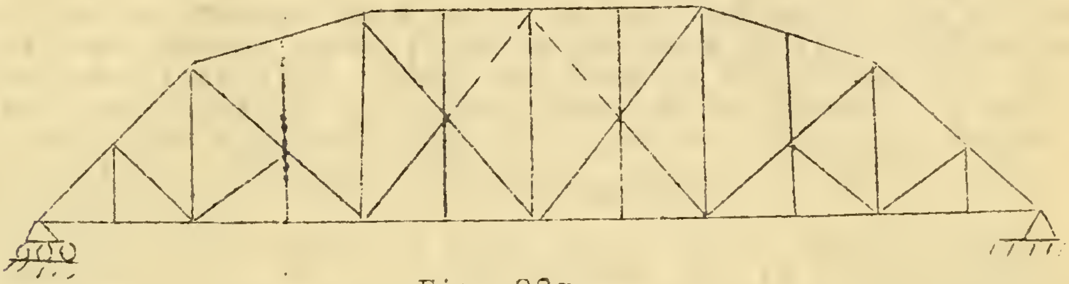

Fig. 98g.
Subdivided Pratt Truss with inclined top chord known as the Petit truss.

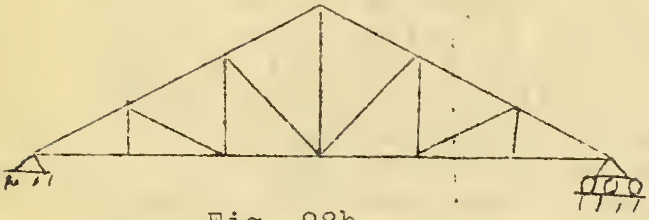

Fig. 98h

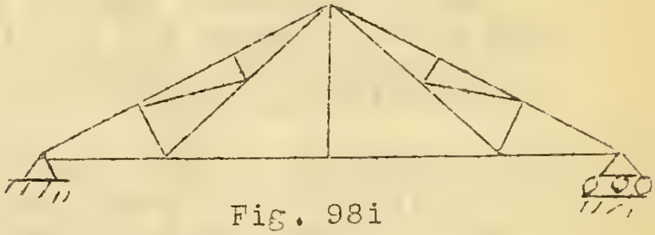

Fig. 98i

Common type of Roof truss.

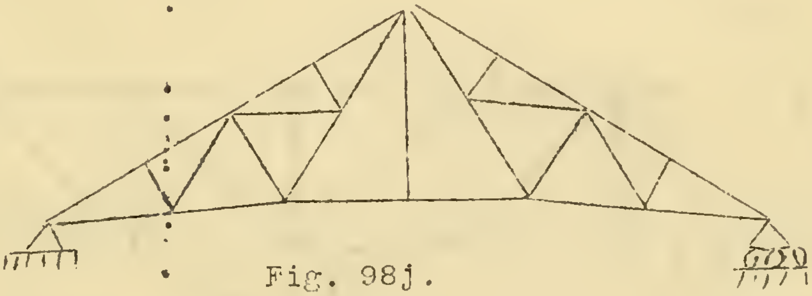

Fig. 98j.

Fig. 98h shows a common type of roof truss which is built both of steel, and of wood with steel verticals. It has no special name but is of the Pratt truss type. Figs. 98i and 98j are Fink roof trusses.

99 BRIDGE TRUSS STRESSES. To completely determine bridge stresses it is desirable to combine the various methods given in the preceding articles. Methods of doing this for the more common types of trusses and for simple loadings are clearly shown in

the numerical examples which follow. Since it is the writer's purpose in this chapter to lay particular emphasis upon truss action rather than upon the consideration of moving concentrated load systems which have already been treated, the live loading used in the examples is taken as a uniform load with a locomotive excess; that is with a single concentrated load equal in magnitude to the excess of the maximum floor beam load due to the actual locomotive over the floor beam load caused by the uniform load. The process of finding the maximum live stress in a member with this loading consists of computing the maximum stress due to the uniform load, and adding to it the maximum stress due to the locomotive excess, this latter being treated as a concentrated load which may be located at any desired panel point regardless of the position of the uniform live load. The dead load is also treated as a uniform load, this being nearly correct for trusses of ordinary span. For trusses of great length or of unusual weight it is better to estimate the actual dead weight acting at each panel point.

It should be remarked that for parallel chord trusses the determination of the live stresses due to concentrated load systems involves merely the computation of the maximum moment at each panel point and the maximum shear in each panel. From the moments the chord stresses may be figured by the method of moments and from the shears the web stresses by the method of shears. If the student thoroughly understands truss action as illustrated in the examples which follow, and the method of using concentrated load systems he should have no difficulty whatsoever in the computation of trusses under concentrated loads.

The fact that this method is used for the determination of truss stresses should not be considered as indicative of the writer's belief that such a method is sufficiently precise for actual use in design. It is used here merely because of its value in showing truss action clearly.

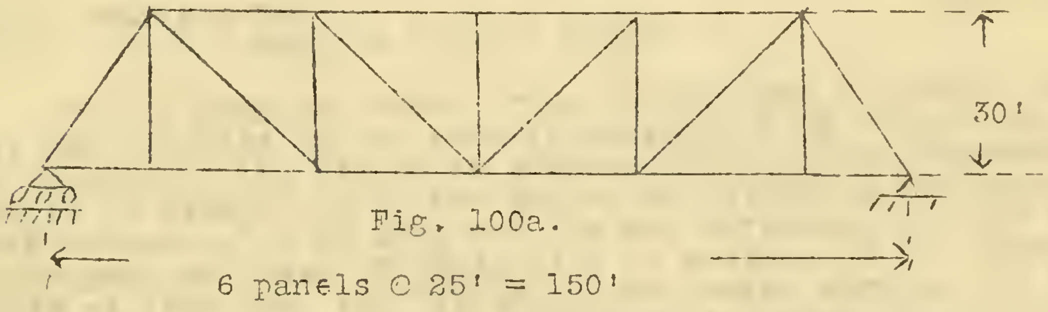

Fig. 100a.

100 COMPUTATION OF STRESSES. PRATT TRUSS. Dead Stresses. For the bridge under consideration let the dead weight be assumed as $W = 1625 = 812$ lbs. per ft. per truss, of which 325 lbs. equals weight of floor and 487 lbs. equals weight of truss. For convenience the weight of floor will be assumed as 300 lb per foot per truss, and of each truss as 500 lbs. per ft. To this weight must be added the weight of the track, that is th

[illegible]

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[illegible]

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rails, ties, spikes, splice bars, guard timbers, etc., which may be assumed as 400 lbs. per linear foot of bridge or 200 lbs. per linear foot of truss, thus giving as a total the following figures:

Weight of floor and track per truss = 500 lbs. per linear ft.
" " one truss 500 lbs. per linear foot.

The weight of the truss will be assumed as applied equally at top and bottom panel points; the weight of the floor is applied at the bottom panel points only since the bridge is a through bridge. The panel loads are, therefore, found to be 250 x 25 = 6250 lbs. on top chord and 750 x 25 = 18750 lbs. on the bottom chord. The stresses in the different members are but little affected by the distribution of the load between the two chords hence assume for ease in computations, that the panel loads on the top chord are 6000 lbs. and on the bottom chord 19000 lbs. The truss will, therefore, be loaded as shown in Fig. 100b.

All chord stresses to be multiplied by 1000 $\frac{P}{h}$
(p=panel length)
(h= height)

Index Stresses

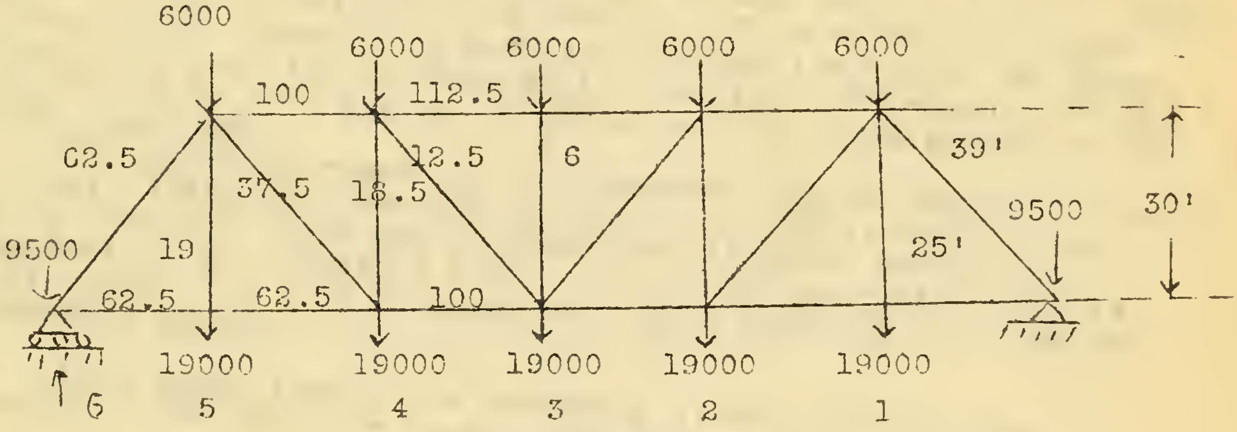

Fig. 100b.

Gross reaction 72000 72000 gross
Net " 62500 = 25000 x 2-1/2

With a parallel chord truss loaded with a uniform load the web stresses may be readily determined by either the method of joints, or the method of shears but the chord stresses should be determined by the method of joints. A combination of both methods will be used for the web stresses, and the vertical components of these stresses will be written on the diagram in units of 1000 lbs. The stress in the centre vertical is first found. This equals 6, and is obtained by using the method of joints. The vertical component in the centre diagonals is next found by the method of shear. It equals 12.5 and can be found without computing the reactions, since truss and loads are symmetrical. The shear in this member due to loads at 1 - 2-4 - and 5 is 0 and the shear due to the load at panel point 3 is one-half that load or one-half of 19+6= $\frac{25}{2}$ = 12.5

[illegible]

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[illegible]

By the method of joints the vertical components in the other web members is readily found to be as shown in Fig. 100b, the stress in the inclined and diagonal, or end post, as it is commonly called, being found to be 62.5 which equals the net reaction thus serving to check the accuracy of all the intermediate figures.

To determine the chord stresses begin at the end of the top chord. The forces acting are as shown in Fig. 100c.

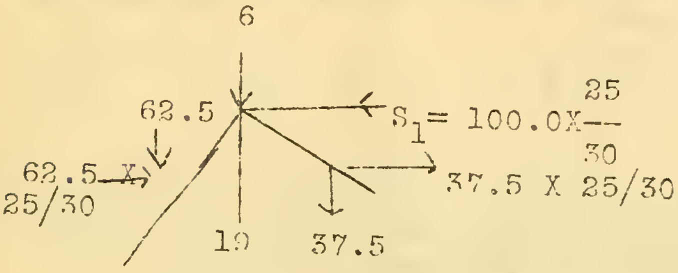


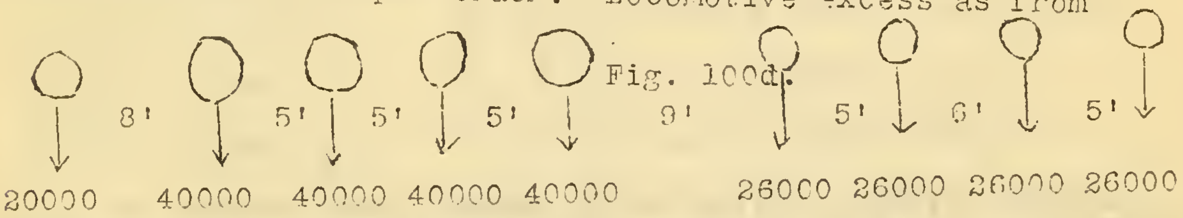
Fig. 100c.

The application of the method of joints gives 100 X 25/30 for the stress in top chord or $100 \text{ X } \frac{p}{h}$ where p = panel length and h = height of truss

By similar methods $112\frac{1}{2}$ x 25/30 is obtained for the stress in the next section of top chord, and 62.5 X 25/30 and 100 X 25/30 for stresses in bottom chord members. This method of determining chord stresses is sometimes called the method of chord increments and the stresses written on the bars which must be multiplied by some factor to obtain actual stresses are called index stresses.

The actual stresses in any diagonal can be easily figured by multiplying its vertical component by its length divided by its height. This can be done for all these members by one setting of the slide rule. This final operation will be left, until the live stresses have been computed in order to economize labor.

Live Load. Assume the following loads: Uniform load 4000 lbs. per foot per track. Locomotive excess as from Fig. 100d.



Loads in thousands of pounds per axle.
Cooper's E40 engine shown in Fig. 100d.

To compute the locomotive excess find the maximum floor beam load caused by the engine. This occurs with the third driving wheel at one of the floor beams as may be seen by applying the criterion for max. moment at the centre of a 50 ft. span and equals.

$$40000\left(\frac{25+20+15+20}{25}\right)+20 \text{ X } 7/25+26 \text{X} \frac{8-1/2}{25} \text{ X } 2 = 151000.$$

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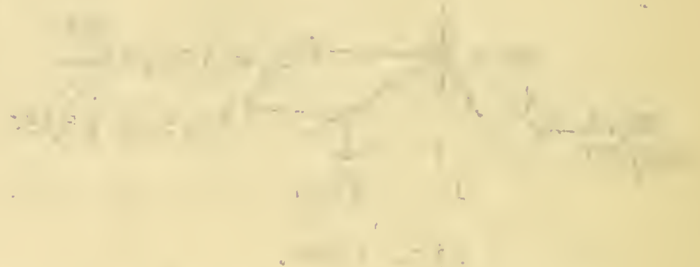

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The panel load produced by the uniform load = 4000 X 25 = 100000 therefore the locomotive excess = 51000 lbs. per panel or for one truss 25500 lbs. For ease in computation, the excess will be taken as 26000 lbs.

Live Stresses. It is evident that the chord stresses will always have their maximum values under a uniform load when that load extends over the entire span since this condition is necessary to produce maximum moments at the panel points, and upon these moments the chord stresses depend. The chord stresses caused by the uniform load, may therefore be obtained by multiplying the dead stresses already found by the ratio of live to dead which in this cast is

$$\frac{2000}{1000}$$

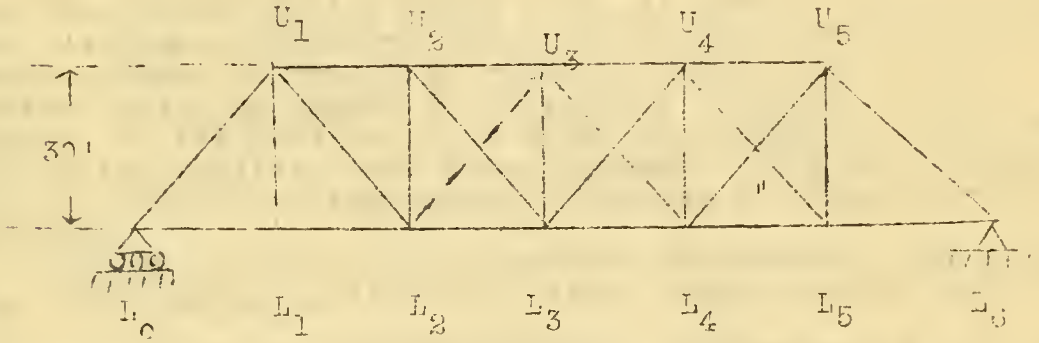


Fig. 100a.

To determine the chord stresses due to the locomotive excess use the method of moments, obtaining the values which follow:

Bar U_1U_2 - E at L_2 - stress = 26 X 4/6 X 50/30 = 28.9 comp.

" U_2U_3 - E at L_3 - " = 26 X 3/6 X 75/30 = 32.5 "

" L_0L_1 - E at L_1 - " = 26 X 5/6 X 25/30 = 18.1 tens.

" L_1L_2 - E at L_1 " = 26 X 5/6 X 25/30 = 18.1 "

" L_2L_3 - E at L_2 " = 26 X 4/6 X 50/30 = 28.9 "

The maximum web stresses occur under a partial loading. To determine their values use the method of shear except for the end vertical and for max. tension load all the panel points to the right of the panel in question with a full load and assume those on the left to be unloaded this being in accordance with the approximate method previously described. The locomotive excess should, of course, be placed so as to increase the stress under consideration as much as possible. The vertical components of the stresses in the web member will then be as follows:

U_1L_2 - Load L_1-L_5 incl. E at L_1 v.c.comp.=15/6 50+5/6 26=146 $\frac{2}{3}$

U_1L_1 " " L_1 " " tens.=50+26 = 76

U_1L_2	Load L_2-L_5 incl.	E at L_2 v.c. tens.=10/6 50+4/6 26 =	$100\frac{2}{3}$
U_2L_2	" L_3toL_5 "	E at L_3 v.c.comp = 6/6 50+3/6 26=	63
U_2L_3-	" L_3toL_5 "	E at L_3v.c.tens= 6/6 50+3/6 26=	63
Counter U_3L_4	" L_4&L_5	E at L_4v.c. " = 3/6 50+2/6 26=	$33\frac{2}{3}$
Counter U_4L_5	" L_5	E at L_5v.c. " = 1/6 50+1/6 26	$12\frac{2}{3}$

Since the truss is symmetrical it is unnecessary to figure the members in the right half. It should be noted, however, that the counter stresses have been computed by figuring the positive shear in panels on right of centre instead of the negative shear in panels to left, this being the simpler method. If the positive live shear in either of these panels exceeds the negative dead shear already computed, counters will have to be used in that panel; otherwise no counters will be required.

One web member has not yet been considered; that is bar U_3L_3. The stress in this will depend upon whether counters are needed in the adjoining panels. If counters are not required there will be no live stress in the member; otherwise the live stress will equal the vertical component in the counter itself. Since the vertical component of the live stress in counter U_3L_4= 33-2/3, and the vertical component of the dead stress = 12-1/2, it is evident that the counter is needed. A similar comparison shows that a counter U_4L_5 is not required. The live stress in U_3L_3 therefore equals the vertical component in U_3L_4= 33-2/3.

Since the counter is in action when this live stress occurs in U_3L_3 its dead stress will be different from that written on the diagram of dead stresses and will equal the vertical component which would exist under dead load in the counter U_3L_4 (assuming the main diagonals L_3U_4 to be out of action) minus the dead panel load on upper chord. This gives a tension equal to 12.5 - 6.0 or 6.5.

Tabulation. This completes the necessary computations except that the actual stresses must be computed from the components which can be done very quickly by the slide rule.

It remains to tabulate the results.

MAXIMUM STRESSES IN THOUSANDS OF POUNDS.

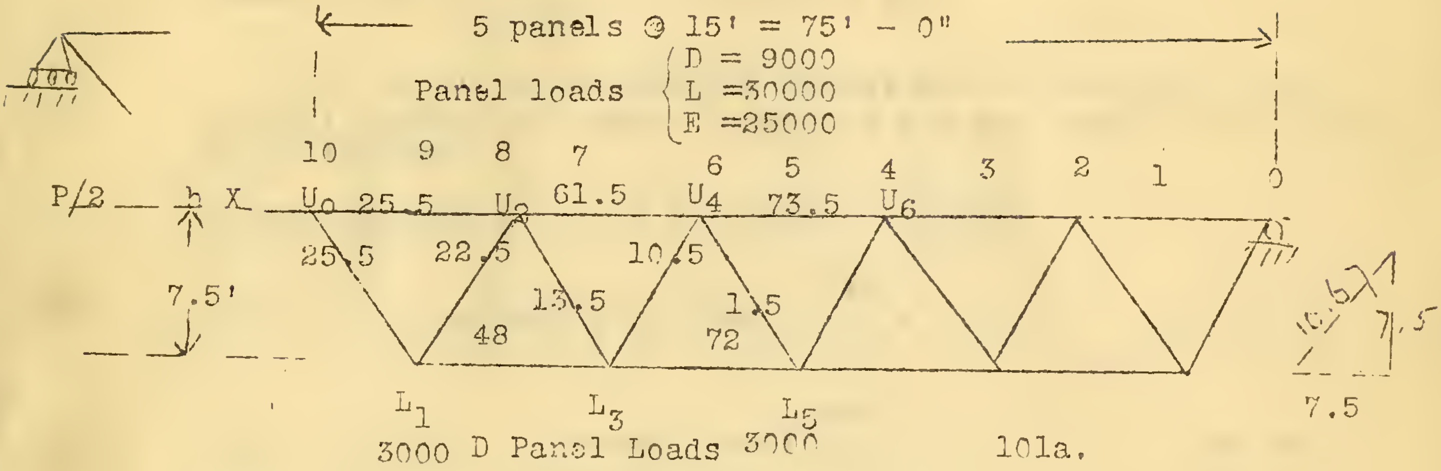
Bar	Dead Stress	Live Stress
U1U2	100 X 25/30 = (-) 83.4	166.8 + 28.9 = (-) 195.7
U2U3	112.5 X 25/30 = (-) 93.8	187.6 + 32.5 = (-) 220.1
L_0L_1	62.5 X 25/30 = (+) 52.1	104.2 + 18.1 = (+) 122.3
L_1L_2	62.5 X 25/30 = (+) 52.1	104.2 + 18.1 = (+) 122.3
L_2L_3	100 X 25/30 = (+) 83.4	166.8 + 28.9 = (+) 195.7
L_0L_1	62.5 X 39/30 = (-) 81.3	146.7 X 39/30 = (-)191.6
U_1L_1	(+) 19	(+) 76
U_1L_2	37.5 X 39/30 = (+) 48.7	100.7 X 39/30 = (+) 131.0
U_2L_2	(-) 18.5	(-) 63
U_2L_3	12.5 X 39/30 = (+) 16.2	63 X 39/30 = (+) 82
*U_3L_3	(+) 6.5	(-) 33.7
*U_3L_4	12.5 X 39/30 = (-) 16.2	33.7 X 39/30 = (+) 43.8

(+) = tension

(-) = compression.

* Stresses in these bars are the dead stresses which would exist when live load is so placed as to bring counter U_3L_4 into action.

101 COMPUTATION OF STRESSES – WARREN TRUSS.



101a.

Assumed Loads – Live load – 2000 lbs. per ft. top chord.
Dead " 600 " " " " "
200 " " " bott "
Excess 25000 " .

Dead Stresses. Index stresses are given on diagram in Fig. 101a.

To check these values compute chord stress in centre of top chord using dead panel loads. Note that for this truss it would be incorrect to compute this stress by figuring the moment produced at the centre, assuming the truss as a girder without floor beams and uniformly loaded, since the moment between floor beams on the top chord is not the same as it would be were there no floor beams.

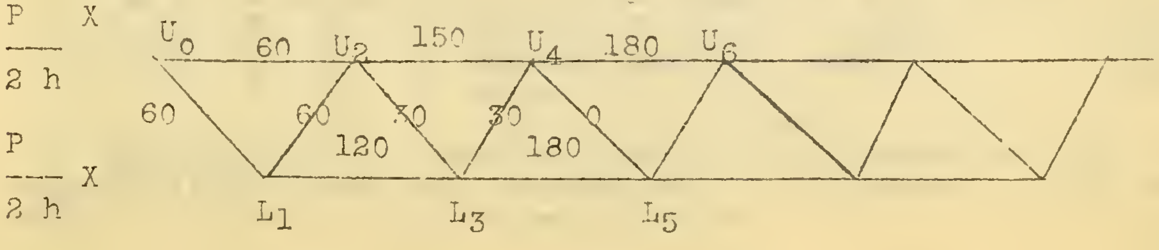
Check. Dead stress – centre panel – top chord
P = panel length

$$= \frac{25.5 \times 2\text{-}1/2\ P - 3 \times 3P - 9 \times 2\ P}{1/2\ P} = 73.5 \quad \text{O.K.}$$

Live Stresses. In this case the live stresses must be computed separately and not obtained by multiplying the dead stresses by the ratio of live to dead load per foot.

The index stresses for the uniform live load for full loading are shown in Fig. 101b.

Fig. 101b.



Check $$\frac{60X2\text{-}1/2\ P - 30\ 2\ P}{1/2\ P} = 180\ \ P \text{ O.K.}$$

The effect of locomotive excess for the chords and of partial loading and excess combined for web members must also be considered.

Stress in chords due to locomotive excess;

UoU2 E @ U_2 stress $= 8/10\ 25\ X\ \frac{7.5}{7.5} =$ 20 (-)

L1L3 E @ U_2 " $= 8/10\ 25\ X\ \frac{2x7.5}{7.5} =$ 40 (+)

U2U4 E @ U_4 " $= 6/10\ 25\ X\ \frac{3\ X\ 7.5}{7.5} =$ 45 (-)

L3L5 E @ U_4 " $= 6/10\ 25\ X\ \frac{4X7.5}{7.5}$ 60 (+)

U4U6 E @ U_6 " $= 4/10\ 25\ X\ \frac{5X7.5}{7.5} =$ 50 (-)

Max. live stresses in web members.

U_0L_1 - V.C. = 60 + 8/10 25 = 80 (+)

U_2L_1 V.C. = " " = 80 (-)

Before computing the remainder of the diagonals it should be noted that inasmuch as this is a Warren truss no counters would be used and hence it is necessary to compute the max. stress of both kinds in diagonals in which tension and compression are both likely to occur.

U2L3 max. live tension V.C. = 12/10 30 + 6/10 25 = 51
or by using the index stresses and removing the load at
U_2 V.C. = 30 + 2/10 30 + 6/10 25 = 51 check.
U2L3 max. live comp, V.C. = 2/10 30 + 2/10 25 = 11
L3U4 V.C.max.live comp.= V.C.max.live tens.in U2L3 = 51
L_3U_4 " " " tens.= " " " comp. " " = 11
U_4L_5 " " " tens.= 6/10 30 + 4/10 25 = 28
" " " " comp.= " " = 28

Tabulation. The dead and max. live stresses or their components have now been determined for all members. The results are shown in the following table:

Maximum Stresses in Thousands of Pounds.

Bar	Dead Stress	Live Stress.
U_0U_2	25.5 (–)	$60 + 20 = 80$ (–)
U_2U_4	61.5 (–)	$150 + 45 = 195$ (–)
U_4U_6	73.5 (–)	$180 + 50 = 230$ (–)
L_1L_3	48.0 (+)	$120 + 40 = 160$ (+)
L_3L_5	72.0 (+)	$180 + 60 = 240$ (+)
UoL1	$25.5 \times \frac{10.6}{7.5} = 36.0$ (+)	$80 \times \frac{10.6}{7.5} = 113$ (+)
U_2L_1	$22.5 \times \frac{10.6}{7.5} = 31.7$ (–)	$80 \times \frac{10.6}{7.5} = 113$ (–)
U_2L_3	$13.5 \times \frac{10.6}{7.5} = 19.1$ (+)	$51 \times \frac{10.6}{7.5} = 72.2$ (+)
U_2L_3		$11 \times \frac{10.6}{7.5} = 15.6$ (–)
L_3U_4	$10.5 \times \frac{10.6}{7.5} = 14.8$ (–)	$51 \times \frac{10.6}{7.5} = 72.2$ (–)
L_3U_4		$11 \times \frac{10.6}{7.5} = 15.6$ (+)
U_4L_5	$1.5 \times \frac{10.6}{7.5} = 2.1$ (+)	$28 \times \frac{10.6}{7.5} = 39.6$ (+)
U_4L_5		$28 \times \frac{10.6}{7.5} = 39.6$ (–)

Stress Diagram. For use in design it is common to show stresses on a diagram of the truss instead of arranging them in tabular form. Such a diagram is called a stress or strain diagram and follows for the truss under consideration. In practice, however, the stress diagram of a bridge would contain in addition to the stresses, the sizes of the truss members, the moments and shears in floor beams and stringers as well as their cross sections, the wind bracing sizes and other similar data.

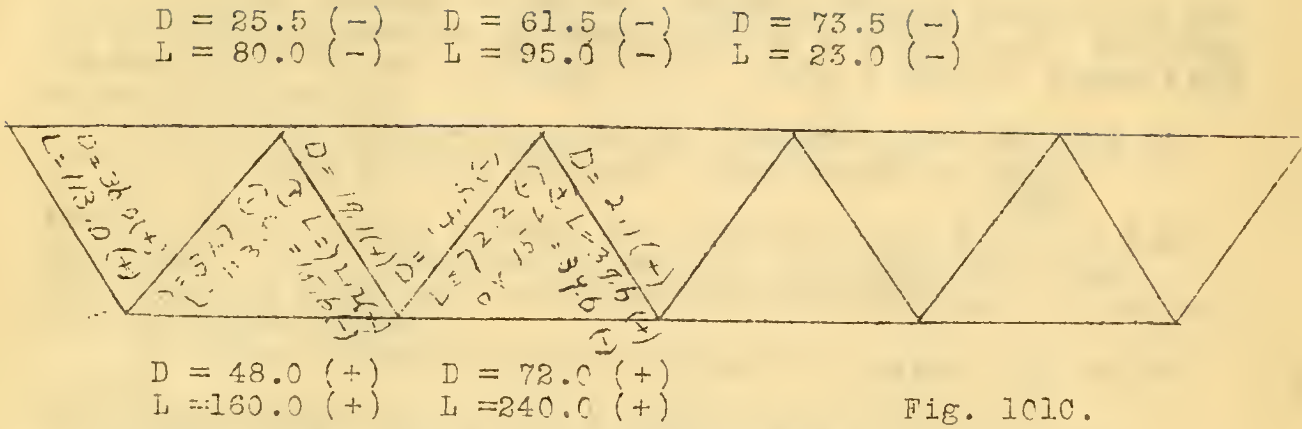


Fig. 101C.

101 COMPUTATION OF STRESSES – SUBDIVIDED WARREN TRUSS. The following example shows the method of computing stresses in a subdivided Warren truss. In this truss the intermediate verticals are used in order that the panel length may be reduced without changing the height of the truss or slope of the diagonals

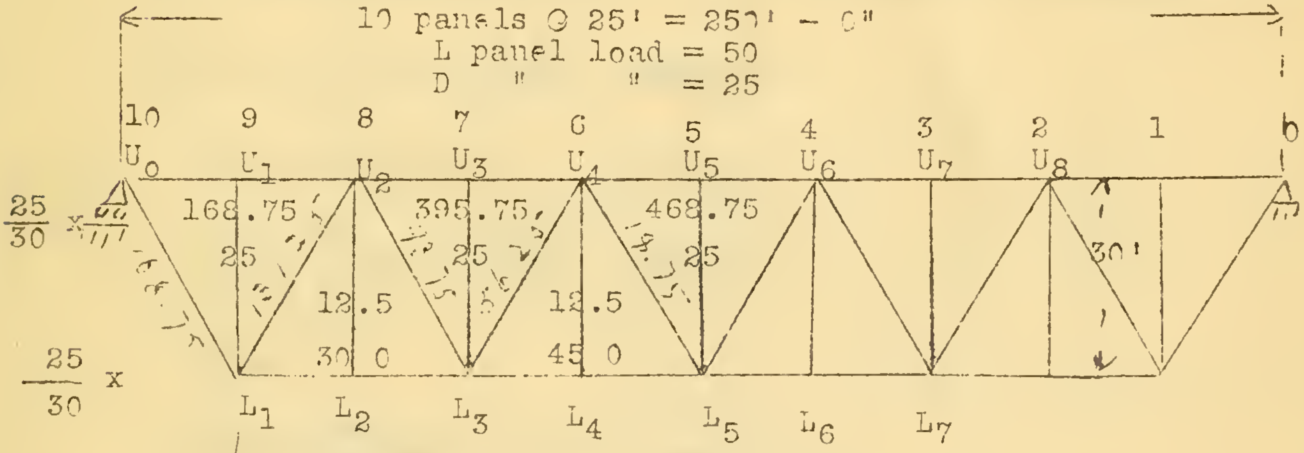


Fig. 102 a.
Index stresses – Dead Loads.

Assumed Dead Load – Top chord 1000 lbs. per ft. Panel load = 25000
" " " Bott. " 500 " " " " "=12500
Live load – Top chord 2000 " " " " "=50000
Loco.Excess " " " "=20000

Dead and Chord Stresses. The index stresses for the dead loads are shown in Fig. 102 a. To check these compute the dead stress in centre member of top chord as follows;–

$$\text{Stress} = 1/8 . \frac{1500}{30} . 250 . 250 = 390625$$

From diagram, stress = 468.75 X 25/30 = 390.62 hence ends and vertical components of all web stresses are correct. To check bottom chord stress note that 450 + 18.75 = 468.75, hence this is also correct.

The chord stresses due to the uniform live load may be computed by multiplying the dead chord stress by $\frac{2000}{1500}$ this being the ratio between the live load per foot and the dead load. This could not be done in the previous truss owing to the panel points in the bottom chord not being under the top panel points.

The chord stresses due to locomotive excess will be as below.

Bar U_0U1 = U1U2 E at U_1 20 X 9/10 X 5/6. = 15 (−)
" L_1L_2 = L2L3 E at U_2 20 X 8/10 X 10/6 = 26 2/3 (+)
" U2U3 = U3U4 E at U_3 20 X 7/10 X 15/6 = 35 (−)
" L_3L_4 = L_4L_5 E at U_4 20 X 6/10 X 20/6 = 40 (+)
" U_4U_5 = U_5U_6 E at U_5 20 X 5/10 X 25/6 = 41 2/3 (−)

Live Web Stresses. The vertical components of the max live web stresses are given by the following equations;

Bar U_0L_1 $\frac{45}{10}\ 50 + \frac{9}{10}\ 20 = 243\ (+)$

" U1L1 $50 + 20 = 70\ (-)$

" U_1U_2 $\frac{36}{10}\ 50 + \frac{8}{10}\ 20 = 196\ (-)$

" U2L2 0

" U2L3 $\frac{28}{10}\ 50 + \frac{7}{10}\ 20 = 154\ (+)$

" L_3U_3 $70\ (-)$

" L3U4 $\frac{21}{10}\ 50 + \frac{6}{10}\ 20 = 117\ (-)$

" L_4U_4 0

" U_4L_5 $\frac{15}{10}\ 50 + \frac{5}{10}\ 20 = 85\ (+)$

" U_5L_5 $70\ (-)$

Bar $L_5U_6 \quad \frac{10}{10}50 + \frac{4}{10}20 = 58\ (-)$ = comp. in U_4L_5

" $U_6L_7 \quad \frac{6}{10}50 + \frac{3}{10}20 = 36\ (+)$ = tension in U_4L_3

" $L_7U_8 \quad \frac{3}{10}50 + \frac{2}{10}20 = 19\ (-)$ = comp. in U_2L_3

Since the truss is symmetrical the stresses in the bars of the right half with the truss loaded from the right equal the stresses in the corresponding bars of the left half with loading from the left, hence in figuring counter stresses, proceed as above, or adopt instead the method used in the previous example, that of loading from the left.

Tabulation. The final results for this truss are shown in the table below.

---MAXIMUM STRESSES IN THOUSANDS OF POUNDS---

Bar	Dead Stress	Live Stress.
U_2U_3 and U_3U_4	$393.75 \times \frac{25}{30} = 328.1(-)$	$328.1 \times 1\frac{1}{3} + 35 = 472.5(-)$
U_0U_1 and U_1U_2	$168.75 \times \frac{25}{30} = 140.6(-)$	$140.6 \times 1\frac{1}{3} + 15 = 202.4(-)$
U_4U_5 and U_5U_6	$468.75 \times \frac{25}{30} = 390.6(-)$	$390.6 \times 1\frac{1}{3} + 41\frac{2}{4} = 562.5(-)$
L_1L_2 and L_2L_3	$300 \times \frac{25}{30} = 250.0(+)$	$250.0 \times 1\frac{1}{3} + 26\frac{2}{3} = 360.0(+)$
L_3L_4 and L_4L_5	$450 \times \frac{25}{30} = 375.0(+)$	$375 \times 1\frac{1}{3} + 40 = 540.0(+)$

Bar	Dead Stress	Live Stress.
U_0L_1	$168.75 \text{ X} \frac{39}{30} = 219.0(+)$	$243 \text{ X} \frac{39}{30} = 316.0(+)$
U_1L_1 U_3L_3 U_5L_5	25 (−)	70.0(−)
L_1U_2	$131.25 \text{ X} \frac{39}{30} = 170.5\ (-)$	$196\text{X} \frac{39}{30} = 235.0\ (-)$
U_2L_2 U_4L_4	12.5 (+)	0.
U_2L_3	$93.75 \text{ X} \frac{39}{30} = 121.9(+)$	$154\text{X} \frac{39}{30} = 200.0(+)$
U_2L_3		$19\text{X} \frac{39}{30} = 24.7(-)$
L_3U_4	$56.25 \text{ X} \frac{39}{30} = 73.1(-)$	$117\text{X} \frac{39}{30} = 152.0(-)$
L_3U_4		$36\text{X} \frac{39}{30} = 46.7(+)$
U_4L_5	$18.75 \text{ X} \frac{39}{30} = 24.4(+)$	$85 \text{ X} \frac{39}{30} = 110.5(+)$
U_4L_5		$58\text{X} \frac{39}{30} = 75.4(-)$

It is evident from above that the diagonal U_4L_5 may have either tension or compression depending upon the position of the live load, but in the other diagonals the live load is insufficient to reverse the dead stress, hence these bars will never have but one kind of stress. Were this a Pratt pin truss, counters would ordinarily be used in panels 4–5 and 5–6.

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